A Robust Design Methodology for Structural Control using Peizoelectrical Materials

by

Syed Jaleeluddin Hyder

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

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This thesis, written by

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Date: 14-7-97

Dedicated to

My

PARENTS,

and

RELATIVES

whose patience and perseverance

led me to this accomplishment.

Acknowledgments

In the name of Allah, Most Gracious, Most Merciful

"Read in the name of thy Lord and Cherisher, who created. Created man from a [leech-like] clot. Read and thy Lord is Most Bountiful. He Who taught [the use] of the pen. Taught man that which he knew not. Nay, but man doth transgress all bounds. In that he looketh upon himself as self-sufficient. Verily, to thy Lord is the return [of all]" (The Holy Quran, Surah 96)

All praise and glory be to Almighty Allah who gave me courage and patience to carry out this work, and peace and blessings of Allah be upon Prophet Muhammad. My heartfelt thanks to my mother and other family members for their blessings, encouragement and moral support. Acknowledge is due to King Fahd University of Petroleum and Minerals for providing support for this research.

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Abstract

Name: Syed Jaleeluddin Hyder

Title: A Robust Design Methodology For Structural Control Using Piezoelectric Materials

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A robust design methodology is presented for the control of flexible structures by the use of piezoelectric actuators. The finite element modeling and analysis of the piezoelectric media are carried out via Hamilton's principle. Finite element equations are utilized for the piezoelectric control of flexible structures subject to dynamic and thermal disturbances.

Various configurations of piezoelectric actuator pairs mounted on cantilever beam structures are considered for illustrative purposes. The dimensions as well as the locations of the actuator pairs are assumed to be varying within certain limits. Taguchi methodology is employed to these piezoelectrically controlled structures as a robust design technique in order to investigate the effects of the design variables on the control performance. A novel optimization problem is also formulated and solved to compare the results with those obtained from Taguchi methodology.

Within the ranges considered in this study, it is concluded that the piezoelectric actuator pairs with larger sizes usually perform better in attenuating the structural vibrations. For the actuator pair location, the actuator pairs close to the fixed end have better performance characteristics.

Master of Science Degree

King Fahd University of Petroleum and Minerals

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ملخص الرسالة

اسم الباحث : سيد حلال الدين حيدر

عنوان البحث : طريقة تصميم منهجية للتحكم فى المنشآت باستخدام مواد ذات احهاد كهربائى

التخصص الرئيسى : الهندسة الميكانيكية

تاريخ الدرجة : يونيو ١٩٩٧

لقد تم تقديم طريقة تصميم قابلة للأستخدام حتى فى الظروف الصعبة للتحكم فى المنشآت المرنة باستخدام محركات احهادية كهربائيةز ولقد تمت النمذجة بطريقة العناصر المحدودة وتحليل الوسط الكهربائى الأحهادى باستخدام مبدأ هاميلتون. كما استخدمت معادلآت العناصر المحدودة للتحكم الأحهادى الكهربائى للمنشآت المرنة المعرضة لأضرابات ديناميكية و حرارية.

ولقد درست عدة أشكال مختلفة من المحركات الأحهادية الكهربائية الأزدواجية موضوعة على عارضة مركبة على شكل كابولى كأمثلة. أبعاد وأماكن اذدواج المحركات افترض تغييرها فى حدود معينة. ولقد تم استخدام طريقة تاحيوشى لدراسة تأثير متغيرات التصميم على أداء منظومة التحكم. كما تم وضع صيغة رياضية حديدة لأ يجاد الحل الأمثل للمشكلة ومقارنة النتائج مع تلك التى تم الحصول عليها بطريقة تاحيوشى.

فى حدود المتغيرات التى تم دراستها فى هذه الرسالة تم استخلاص ان أزواج المحركات الأحهادية الكهربائية ذات الأحجام الأكبر أدائها عادة أفضل فى تقليل اهتزاز المنشآت أما بالنسبة لمكان أزواج المحركات فان المحركات الأقرب للطرف المثبت لها خواص أداء أفضل.

درحة الماحستير فى العلوم

حامعة الملك فهد للبترول و المعادن

الظهران-المملكة العربية السعودية

يونيو ١٩٩٧

Nomenclature

a_j	Generalized Coordinate
ANOVA	Analysis of variance
B_{ss}	Thickness of the piezoelectric actuator pairs, m
c	Matrix of elastic stiffness coefficients, Pa
c_p	Capacitance, F
$C_{\theta u}$	Thermal expansion rate matrix
$C_{\theta\phi}$	Pyroelectric rate matrix
$C_{\theta\theta}$	Heat conduction rate matrix
d	Length of the piezoelectric actuator pairs, m
df	Degrees of freedom of each factor
df_e	Degrees of freedom of error
D	Electric displacement vector
e	Piezoelectricity matrix
E	Electric field vector
f	Objective function, m
F	Global force vector
G	Potential

$\mathbf{G}$	Global charge vector
$\mathbf{h}$	Vector of heat flux
h_b	Height of the structure, m
K_i	Kinetic energy, J
K_{uu}	Elastic stiffness matrix
$K_{u\phi}$	Piezoelectric stiffness matrix
$K_{\phi\phi}$	Dielectric matrix
$K_{u\theta}$	Thermal expansion stiffness matrix
$K_{\phi\theta}$	Pyroelectric matrix
$K_{\theta\theta}$	Heat conduction matrix
L	Length of the structure, m
L_u	Differential operator matrix
M_{uu}	Mass matrix
MSD	Meansquare Deviation
N_u	Displacement shape function matrix
N_ϕ	Electrical shape function matrix
N_θ	Thermal shape function matrix
OA	Orthogonal array
$\mathbf{P}$	Pyroelectric coefficient vector
$\mathbf{P_b}$	Body force vector

$\mathbf{P_c}$	Concentrated force vector
$\mathbf{P_s}$	Surface force vector
$\mathbf{Q}$	Heat vector
$\mathbf{S}$	Strain vector
S_x	Sum of squares
S_e	Sum of squares of the error
S/N	Signal-to-noise ratio
$\mathbf{T}$	Stress vector
tr(P)	Trace of P
tr(Q)	Trace of Q
$\mathbf{u}$	Global displacement vector
V	Applied voltage
V_x	Variance of each factor
V_e	Variance of error
W_s	Width of the piezoelectric actuator pairs, m
Y_b	Young's modulus for the structure, Pa
Z	Distance of the piezoelectric actuator pairs from the fixed end, m
ϵ	Dielectricity matrix
ϕ	Global electric potential vector
ν	Poisson's ratio

ρ	Mass density, $\frac{kg}{m^3}$
σ	Surface charge, C
θ	Global temperature variation vector

Chapter 1

Introduction

Due to increasing demands of high structural performance requirements, the modelling and control of large flexible structures have attracted considerable amount of researchers in recent years. The rapid developments in space exploration and robotics have reached a level which calls for a departure from conventional control approaches in order to satisfy the stringent system performance requirements. As a result control studies of flexible structures have coursed into new channels to design controlled structures with high performance characteristics.

There are two basic steps involved in the modeling and control of a flexible structure using numerical techniques. In the first step the structure is usually modeled using Finite Element Method (FEM) which results in large number of Ordinary Differential Equations (ODE). The locations of sensors and actuators for the structure must be either assumed or determined by some scheme. In the second step the dynamic equations of motion are used to determine a control law appropriate for the structure.

Piezoelectric materials have been used at an increasing rate for behavior sensing and control of flexible structures. Due to their distinct nature, these materials can be used for distributed motion sensing and monitoring of distributed parameter systems, such as beams, plates and shells. The flexible structural systems employing piezoelectric sensors and actuators are often called intelligent or smart structures owing to their highly adaptive capabilities.

1.1 Piezoelectricity

When processed under certain conditions, some crystals and polymers exhibit the phenomenon of coupling between mechanical and electrical fields. This phenomenon is known as piezoelectricity which has been the subject of many research activities and applications over a century after its discovery by Curie brothers [1]. Piezoelectricity is the phenomenon used mostly in the distributed sensing and actuation of flexible structures. Piezoelectricity relates electric polarization to mechanical stress/strain in piezoelectric materials. It basically defines a relation between mechanical and electric fields in certain piezoelectric ceramics, films and crystals. In addition to being bonded to the surfaces of beams, plates, shells etc., piezoelectric materials can be embedded in these structural elements. The advantage of embedding is the better mechanical and electrical communication between the piezoelectric material and main structure. The disadvantage may be the complication in manufacture. Some advantages of piezoelectric materials are as follows:

- suitable for measuring micron level displacements,
- low power for operation,
- low energy consumption,
- no need for thermal cooling, and
- ease of operation in flexible structures.

There are two basic phenomena which permit piezoelectric materials to be used as sensors and actuators. These are as follows :

1. Direct effect: An electric charge is observed when a piezoelectric material is deformed [2, 3, 4].
2. Converse effect: Application of an electric field to the piezoelectric material introduces mechanical stress/strain [5, 6, 7].

In modern days piezoelectricity has found applications in many applications owing to its easy availability, ease of handling and accurate structural sensing and control performances. There are several piezoelectric materials, but most prominent among them are: lithium niobate ($LiNbO_3$), lead zirconate titanate (PZT), and polyvinylidene fluoride (PVDF or $PVDF_2$).

1.2 Thermopiezoelectricity

It has long been recognized that mechanical, electrical and thermal fields are coupled in most of the physical problems. Due to their inherent complexity, relatively few solutions to such coupled problems are available in the literature.

The term "thermopiezoelectricity" in this study is used for the inclusion of thermal effects in piezoelectric media. Hence the thermopiezoelectricity is the research study of the coupled effects of mechanical, electrical and thermal fields in piezoelectric materials. The piezoelectric sensors and actuators under thermal influences are sometimes referred as thermopiezoelectric sensors and actuators. Although the theory of thermopiezoelectricity has been well-established, its use in vibration control of flexible structures has been considered only in the last few years.

1.3 Taguchi Methodology

The challenge to produce high-quality products and continue to improve the quality under scarcity of good-quality raw material, high-quality manufacturing equipment and skilled engineers exists for many manufacturers today. The task of developing a methodology to meet the above challenge led Taguchi to develop a robust design methodology called after his name [8, 9, 10]. Through his research in the 1950's and the early 1960's, Taguchi developed the foundations of robust design and validated its basic philosophies by applying them in the development of many products.

Taguchi's methodology involves the use of only a fraction of the possible combination of all the factors affecting the objective function. The selection of these combinations, and the generation and manipulation of arrays is presented by Taguchi through the aid of Orthogonal Array (OA)'s [11]. Later on, a data-analysis technique known as Analysis of Variance (ANOVA) is used to find the effect of each factor on the objective function.

1.4 Motivation

Piezoelectric materials have been used with an increasing rate in sensing and control of flexible structures. The direct and converse piezoelectric effects allow piezoelectric materials to be used as distributed sensors and actuators, respectively. Distributed sensing and actuation phenomena lead to better monitoring and control of systems, as compared with conventional discrete sensing and actuation.

It is always a challenge to design the systems and products such that their performance characteristics are within specified range. Taguchi methodology enables the system design to be robust and allows the designer to determine the factors affecting the system performance with the smallest number of simulations. Hence, Taguchi methodology is used to perform a robust study on the control of flexible structures using piezoelectric actuators. The motivation is to determine the control parameters associated with the sizes and locations of piezoelectric actuators and to observe their effects on the control performance through Taguchi methodology. Various objective functions are used to represent the control performance of the piezoelectric actuators bonded to the flexible structures. Suitable OA's are chosen and simulations are performed to achieve the best control performance within specified limits.

1.5 Objectives

In this proposed work the robust design Taguchi methodology is used to observe the effects of variation of length, width, thickness and location of piezoelectric actuators on the control of structural systems and to design the actuators so as to obtain the best control performance.

Control performance is measured through the vibration simulations of the piezoelectric system consisting of a cantilever beam structure and piezoelectric actuator pairs. The magnitudes of the oscillations in a given period are summed which is termed as objective function. Also considered as objective function is the effective damping time of structural vibrations. The smaller the value of objective function the better is the control performance. The vibrations of the beam is controlled by the linear quadratic regulator (LQR) control method using the piezoelectric actuators. The vibrations of the system are assumed to be due to vertical step/impulse forces at the tip of the beam. Taguchi methodology is applied to the system for the purpose of studying the effects of the location, length, width and thickness of the piezoelectric actuators on the control performance.

In brief, following objectives are considered:

1. The effects of the location, length, width and thickness of the piezoelectric actuators on control performance using FEM and Taguchi methodology are investigated.

2. The thermal effects are then added to the finite element model of the cantilever beam and piezoelectric actuators, and Taguchi robustness is studied on the control performance of the piezoelectric actuators mounted to the cantilever beam structure.

1.6 Outline of Thesis

In this work, the cantilever beam-like flexible structures mounted with piezoelectric actuator pairs (piezoelectric control systems) are considered for the robust design and analysis using Taguchi methodology. Chapter 1 is the introductory chapter. In Chapter 2, literature review pertaining to the research of sensing and control of systems by piezoelectric structures and Taguchi methodology is discussed in brief.

In Chapter 3, FEM is used for modeling the coupled mechanical (structural) and electrical field equations governing the behavior of piezoelectric media. Then steady temperatures are imposed on the structure to study the effect of temperature variations on the response. In Chapter 4, basic principles of Taguchi methodology and its application to piezoelectric materials are discussed. In Chapter 5, three different structure-piezoelectric actuator pair configurations are used for Taguchi robust design and analysis. A standard constrained optimization problem is also set in order to minimize the objective functions subject to constraints on the sizes of finite elements. The goal in setting the optimization problem is to compare the results with those obtained from Taguchi methodology. In Chapter 6, two configurations are used for temperature variation analysis. The aim in these case studies is to increase the control performance of the piezoelectric actuators when subjected to mechanical and thermal disturbances. Depending upon the configuration, the design variables affecting the control performance are taken as actuator dimensions and locations assumed to be varying within certain limits. The signal-to-noise ratio (S/N) and ANOVA are evaluated and the optimal levels of design variables are found for each configuration.

Chapter 2

Literature Review

The remarkable advances in structural design and control have made the piezoelectric materials the central components of intelligent structures, the structures with highly adaptive capabilities [12]. The direct and converse piezoelectric effects of distributed nature enable these materials to be used as distributed sensors and actuators, respectively. The polyvinylidene fluoride (PVDF) is a polymeric piezoelectric film discovered by Kawai [13]. It is highly regarded for its flexibility, durability and light weight. The mechanics of spatially distributed piezoelectric shell convolving sensors was studied by Tzou et al [2]. The sensors were shaped to prevent the observation spillover problem. Piezoelectric sensors embedded into a cantilever beam and a filament-wound cylinder were used by Cheng et al [14] for sensing vibration and acoustic emissions.

The use of piezoelectric materials as actuators is the consequence of converse piezoelectric effect. Piezoelectric actuators are often embedded or bonded to beam, plate and shell structures for mode actuation and/or attitude control. The swept-forward wing structures were modeled by Song et al [15] as thin-walled beams and the static aeroelastic behavior control was considered by incorporating piezoelectrics. The behavior of piezo-actuated laminated plates was analytically investigated by Zhou and Tiersten [16]. A simultaneous piezoelectric sensor/actuator is a device which can function both as a sensor and actuator. The modeling and implementation of such a device were considered by Anderson and Hagood [3]. The slewing control and vibration suppression of a slewing flexible structure were carried out by Denoyer and Kwak [17] employing piezoelectric sensors and actuators.

The FEM is established as a powerful numerical technique which provides solutions to many complicated engineering problems and is widely used in modern engineering designs and analyses. The distributed sensing phenomena of a PVDF shell were investigated by Tzou and Tseng [4, 18] using the FEM. In the piezoelectric sensor/actuator design study of distributed parameter systems by Tzou and Tseng [19], a new piezoelectric finite element with internal degrees of freedom (DOF) was developed. After condensation of the internal DOF into the physical DOF, the standard brick elements were used for the finite element modeling of piezoelectric layers. Since these brick elements had the internal degrees of freedom embedded within, they had good response characteristics suited for thin piezoelectric layers. After finite element modeling of the master structure (plate element) and piezoelectric layers, two control laws were implemented for vibration suppression of the master structure, namely constant gain feedback control and negative velocity constant amplitude feedback control.

The mechanical and electrical responses of laminated composites containing piezoceramics acting as sensors and actuators were analytically studied by Ha and Chang [20]. A finite element analysis based on the theory of elasticity for anisotropic and inhomogenous materials combined with piezoelectricity was developed to simulate the response of the structures subject to mechanical and electrical loads. A finite element simulation of the dynamic responses of piezoelectric motors and actuators was presented by Kagawa et al [21] by considering the time responses of three-dimensional (3-D) piezoelectric structures. The finite element formulation of piezoelectric media was presented by Sunar and Rao [22] for piezoelectric control of flexible structures subjected to structural vibrations. The effectiveness of piezoelectric actuators was studied for various actuator locations.

Thermopiezoelecticity, piezoelectricity with thermal effects, is long studied and its theory is well-established. The governing equations of a thermopiezoelectric medium were first derived by Mindlin [23]. The application of thermopiezoelectricity in distributed sensing and control of flexible structures has drawn attention in recent years [24, 25]. It is recognized that the presence of a thermal field may adversely affect the control performance and hence it needs to be included for precision control.

In the area of structural design, numerous efforts have been devoted to weight reduction and strength enhancement as well as accomplishing an automatic design methodology. Robust Design is one of the ways to satisfy these conflicting conditions. By robust, we mean that the product or process performs consistently on target and is relatively insensitive to factors that are difficult to control. Hence, the robust design process is necessarily concerned about finding the parameters which have impact on the design and the degree of impact. Being one of the techniques of the robust design, Taguchi methodology can be applied to a wide variety of problems [26, 27, 28]. The key component of Taguchi's philosophy of robust design is the reduction of variability around its target value. The three basic steps of Taguchi methodology are named as concept or system design, parameter design and tolerance design [29]. Taguchi methodology of robust design includes reduction of noise and cost in the design stage which is termed as parameter design. The existing robust design technology implementing the Taguchi method uses the analysis of variance (ANOVA) on the signal-to-noise ratio (S/N) to determine the optimum levels of design variables.

The use of the Taguchi method as a means to identify the optimum design of a robot sensor was done by Rowlands and Pham [27]. The Taguchi method was employed as a systematic method to understand the performance of the sensor while using a limited number of model evaluations. The Taguchi method was based on the experimental design technique and the results of using a full factorial design and fractional factorial design were compared. A 3-D nonlinear finite element model of a molded plastic ball grid array was developed by Mertol [28] using finite element simulations. The model was used to optimize the package for robust design and to determine design rules to keep package waurpage within acceptable limits.

The Taguchi method was used by Park et al [30] for discrete structural design to enhance the performances of the final design. An orthogonal array was constructed with discrete values around an optimal solution and the cost function was evaluated by the Taguchi method. Excellent results were obtained for the constrained problems and fairly good designs were achieved for the unconstrained problems.

Chapter 3

Modelling of Piezoelectric/ Thermopiezoelectric Media

3.1 Introduction

In the last several years, with the advent of computers the FEM have gained considerable prominence owing to its applications in diverse fields. The FEM is one of the numerical methods that can solve boundary value problems, initial value problems and eigenvalue problems. Currently this method is one of the numerical tools applied to the analysis of piezoelectric materials.

A general piezoelectric control system with sensor and actuator is shown in Fig. 3.1. The piezoelectric systems used in this thesis are depicted in Fig. 3.2, where the rectangular finite elements for both the beam and piezoelectric actuators are indicated. The rectangular finite elements for piezoelectric actuators contain pseudo internal DOF. The internal DOF are for better representation of bending moments generated by the piezoelectric actuators and are condensed into the physical DOF using Guyan reduction technique [31].

The piezoelectric control system shown in Fig. 3.1 consists of a main structure, a piezoelectric sensor layer at the bottom of the structure and a piezoelectric actuator layer at the top of the structure. The bottom piezoelectric layer (piezoelectric sensor) senses the displacement of the beam and generates voltage in response to the beam displacement. This voltage is multiplied by some gain according to the control law implemented and is fed back to the top piezoelectric layer (piezoelectric actuator). The top layer reacts to the feedback voltage and generates mechanical motion. This motion can be made to oppose the motion of the beam if the feedback voltage is applied 180 degrees out of phase and hence the vibration attenuation of the beam can be effectively achieved.

As mentioned before, the modeling and analysis of thermopiezoelectric sensors and actuators are carried out using the FEM. Cantilever beams with piezoelectric actuator pairs are considered as case study problems for the piezoelectric control (Fig. 3.2). Mechanical and steady thermal disturbances are imposed on the beam and vibrational displacements are computed, and the absolute sum of these vibrational displacements is selected as one of the objective functions in minimizing the vibrational oscillations. Another objective function is chosen as the effective damping time of structural oscillations.

3.2 Finite Element Formulation of Thermopiezoelectric Problems

The governing equations for thermopiezoelectricity are given by [23]

$$
\begin{aligned}
\mathbf{T} &= c\mathbf{S} - e\mathbf{E} - \lambda\theta \\
\mathbf{D} &= e^T\mathbf{S} + \epsilon\mathbf{E} + \mathbf{P}\theta \\
\eta &= \lambda^{\mathbf{T}}\mathbf{S} + \mathbf{P}^T\mathbf{E} + \alpha\theta
\end{aligned}
\tag{3.1}
$$

The following relations are also used

$$
\mathbf{h} = -K\nabla\theta, \qquad \mathbf{E} = -\nabla\phi \tag{3.2}
$$

In the above equations **T**, **S**, **D** and **E** are the vectors of stress, strain, electrical displacement and electrical field. ϕ, θ and η are the electric potential, temperature change and entropy per unit volume. The generalized heat equation is given by

$$
\Theta\dot{\eta} = -\nabla^{\mathbf{T}}\mathbf{h} + W \tag{3.3}
$$

where W is the heat source per unit volume, and

$$
\Theta = \theta_0 + \theta
$$

where θ_0 is the reference temperature.

By using Eqs.(3.1) through (3.3) the finite element approximation of the heat equation is obtained as [24]

$$-C_{\theta u}\dot{\mathbf{u}} + C_{\theta\phi}\dot{\phi} - C_{\theta\theta}\dot{\theta} - K_{\theta\theta}\theta = \mathbf{Q} \tag{3.4}$$

The element matrices and external heat vector are given by

$$\begin{aligned}
C_{\theta ue} &= \int_{V_e} \theta_0 N_\theta^T \lambda^{\mathbf{T}} B_u dV \\
C_{\theta\phi e} &= \int_{V_e} \theta_0 N_\theta^T \mathbf{P}^{\mathbf{T}} B_\phi dV \\
C_{\theta\theta e} &= \int_{V_e} \theta_0 N_\theta^T \alpha N_\theta dV \\
K_{\theta\theta e} &= \int_{V_e} B_\theta^T K^T B_\theta dV \\
\mathbf{Q_e} &= \int_{A_e} N_\theta \mathbf{h}^{\mathbf{T}} \mathbf{n} dA - \int_{V_e} W N_\theta^T dV
\end{aligned} \tag{3.5}$$

where V_e is the volume of the element considered, A_e is the element area whose normal vector is $\mathbf{n}$, $B_u = L_u N_u$ where L_u is the differential operator matrix, $B_\phi = \nabla N_\phi$ and $B_\theta = \nabla N_\theta$. In the above equations, the subscripts u, ϕ and θ denote the mechanical field, electrical field and thermal field, respectively. Furthermore, the Hamilton's principle is given by

$$\delta \int_{t_1}^{t_2} (K_i - \Pi) dt = 0 \tag{3.6}$$

K_i in the above equation is the kinetic energy defined as

$$K_i = \frac{1}{2} \int_V \rho \dot{\mathbf{u}}^{\mathbf{T}} \dot{\mathbf{u}} dV \tag{3.7}$$

In Eq. 3.6, the functional Π is given by [32]

$$\Pi = \int_V (G + \eta\Theta) dV - \int_V \mathbf{u}^T \mathbf{P_b} dV - \int_{S_1} \mathbf{u}^T \mathbf{P_s} dS - \mathbf{u}^T \mathbf{P_c} + \int_{S_2} \phi\sigma dS \qquad (3.8)$$

where G is the thermodynamic potential, $\mathbf{P_b}$ is the vector of body forces applied to volume V, $\mathbf{P_s}$ is the vector of surface forces applied to surface S_1, $\mathbf{P_c}$ is the concentrated load vector, and σ is the surface charge on the surface S_2. Virtual changes in Hamilton's principle are given by

$$\delta \int_{t1}^{t2} K_i dt = \int_{t1}^{t2} \int_V \rho\delta\dot{\mathbf{u}}^T \dot{\mathbf{u}} dV dt = - \int_{t1}^{t2} \int_V \rho\delta\mathbf{u}^T \ddot{\mathbf{u}} dV dt \qquad (3.9)$$

and

$$\delta \int_{t1}^{t2} \Pi dt = \int_{t1}^{t2} \left[\delta \int_V G dV - \delta \int_V \mathbf{u^T P_b} dV - \delta \int_{S1} \mathbf{u}^T \mathbf{P_s} dS - \delta\mathbf{u}^T \mathbf{P_c} + \delta \int_{S2} \phi\sigma dS \right] dt \qquad (3.10)$$

where the virtual change in potential G is given by

$$\delta G = \delta\mathbf{S}^T \mathbf{T} - \delta\mathbf{E}^T \mathbf{D} - \delta\theta\eta \qquad (3.11)$$

The following relations are defined for the finite element formulation of a piezoelectric material subjected to structural and thermal disturbances. Let

$$\mathbf{u_e} = N_u \mathbf{u_i}$$

$$\phi_e = N_\phi \phi_{\mathbf{i}}$$

$$\theta_e = N_\theta \theta_i \qquad (3.12)$$

where N_u, N_ϕ and N_θ are the shape function matrices for mechanical, electrical and thermal fields, and $\mathbf{u_i}$, ϕ_i and θ_i are the vectors of nodal displacement, electric potential and temperature variation. The subscript e in the equations stands for the element. Relating strain to displacement, and electric field to electric potential yields

$$\mathbf{S_e} = L_u \mathbf{u_e} = [L_u N_u] \mathbf{u_i} = B_u \mathbf{u_i}$$

$$\mathbf{E_e} = -\nabla \phi_e = -[\nabla N_\phi] \phi_\mathbf{i} = -B_\phi \phi_\mathbf{i} \tag{3.13}$$

where L_u is a differential operator matrix.

$$L_u = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \tag{3.14}$$

Substituting the above equations in Eq.(3.6) gives the equation of motion

$$M_{uu}\ddot{\mathbf{u}} + K_{uu}\mathbf{u} + K_{u\phi}\phi - K_{u\theta}\theta = \mathbf{F}$$

$$K_{\phi u}\mathbf{u} - K_{\phi\phi}\phi + K_{\phi\theta}\theta = \mathbf{G} \tag{3.15}$$

where the element matrices and vectors are given by

$$M_{uue} = \int_{V_e} \rho N_u^T N_u dV, \qquad K_{uue} = \int_{V_e} B_u^T c B_u dV$$

$$K_{u\phi e} = \int_{V_e} B_u^T e B_\phi dV, \qquad K_{u\theta e} = \int_{V_e} B_u^T \lambda N_\theta dV$$

$$K_{\phi ue} = \int_{V_e} B_\phi^T e^T B_u dV, \qquad K_{\phi\phi e} = \int_{V_e} B_\phi^T \epsilon B_\phi dV$$

$$K_{\phi\theta e} = \int_{V_e} B_\phi^T \mathbf{P} N_\theta dV$$

$$G_e = -\int_{S_{2e}} N_\phi^T \sigma dS \tag{3.16}$$

$$\mathbf{F_e} = \int_{V_e} N_u^T \mathbf{P_b} dV + \int_{S_{1e}} N_u^T \mathbf{P_s} dS + N_u^T \mathbf{P_c}$$

The heat, actuator and sensor equations are written in the following form

$$\begin{bmatrix} M_{uu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\phi} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -C_{\theta u} & C_{\theta\phi} & -C_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\phi} \\ \dot{\theta} \end{Bmatrix}$$

$$+ \begin{bmatrix} K_{uu} & K_{u\phi} & -K_{u\theta} \\ K_{\phi u} & -K_{\phi\phi} & K_{\phi\theta} \\ 0 & 0 & -K_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \phi \\ \theta \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{G} \\ \mathbf{Q} \end{Bmatrix} \tag{3.17}$$

For a rectangular element of the size (2a $\times$ 2b) with W_s as the width the shape functions are given as

$$N_1 = \frac{1}{4}(1 - \frac{x}{a})(1 - \frac{y}{b})$$

$$N_2 = \frac{1}{4}(1 + \frac{x}{a})(1 - \frac{y}{b}) \tag{3.18}$$

$$N_3 = \frac{1}{4}(1 + \frac{x}{a})(1 + \frac{y}{b})$$

$$N_4 = \frac{1}{4}(1 - \frac{x}{a})(1 + \frac{y}{b})$$

Internal DOF are added into the element to give a better representation to the bending moments caused by the piezoelectric effects. Two shape functions are defined for this purpose which are given as follows

$$N_5 = \frac{a^2 - x^2}{a^2}$$
$$N_6 = \frac{b^2 - y^2}{b^2}$$

These shape functions vanish at the element boundaries when $x = \pm a$ and $y = \pm b$. The displacement vector, $\mathbf{u_e}$, is now expressed as

$$\mathbf{u_e} = N_u \mathbf{u_i} + X \mathbf{a_j} \tag{3.19}$$

The strain vector, $\mathbf{S_e}$, is now written as [19]

$$\mathbf{S_e} = B_u \mathbf{u_i} + Y \mathbf{a_j} \tag{3.20}$$

where $\mathbf{a_j}$ is the added generalized coordinate vector and X in the above equation is given by

$$X = \begin{bmatrix} 0 & 0 \\ N_5 & N_6 \end{bmatrix} \tag{3.21}$$

and Y is given as

$$Y = L_u X = -2 \begin{bmatrix} 0 & 0 \\ 0 & y \\ x & 0 \end{bmatrix} \tag{3.22}$$

Equation (3.17) is modified to include the effect of the new pseudo internal DOF as

$$
\begin{bmatrix} M_{uu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\phi} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -C_{\theta u} & C_{\theta\phi} & -C_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\phi} \\ \dot{\theta} \end{Bmatrix}
$$

$$
+ \begin{bmatrix} K^{*}_{uu} & K_{u\phi} & -K_{u\theta} \\ K_{\phi u} & -K_{\phi\phi} & K_{\phi\theta} \\ 0 & 0 & -K_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \phi \\ \theta \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{G} \\ \mathbf{Q} \end{Bmatrix} \tag{3.23}
$$

where Guyan static model reduction technique is used to condense the pseudo internal DOF to the physical nodal DOF. The new matrix, K^{*}_{uu}, is the global elastic stiffness matrix which is assembled of $[K_{uu}]^{*}_{e}$ given by

$$
[K_{uu}]^{*}_{e} = [K_{uu}]_e - [K_{ua}]_e [K_{aa}]^{-1}_{e} [K_{au}]_e \tag{3.24}
$$

where $[K_{ua}]_e$ and $[K_{aa}]_e$ are partitioned stiffness matrices given by

$$
[K_{ua}]_e = \int_V B_u^T c Y dV \tag{3.25}
$$

$$
[K_{aa}]_e = \int_V Y^T c Y dV
$$

The heat generation in a distributed control system may be due to high voltage applied to the piezoelectric actuator and/or the high or low temperature environment in which the control system is to operate. In all the above cases Eq. (3.23) can be used to study the thermal effects on the distributed control system.

3.3 Finite Element Formulation of Piezoelectric Equations

The linear piezoelectric equations without thermal effects are written as [33]

$$\mathbf{T} = c\mathbf{S} - e\mathbf{E}$$

$$\mathbf{D} = e^T\mathbf{S} + \epsilon\mathbf{E} \tag{3.26}$$

Finite element formulation of piezoelectric media is carried out using Hamilton's principle given by Eq. (3.6), where K_i and Π are given by the functional given by the Eqs. (3.7) and (3.8). Note that in this case, the functional Π contains no thermal field. Virtual changes in Hamilton's principle are given by Eqs. (3.9) and (3.10). Using the above equations in Hamilton's principle and neglecting the thermal effects yields the finite element equations which are collectively written as

$$\begin{bmatrix} M_{uu} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} K^*_{uu} & K_{u\phi} \\ K_{\phi u} & -K_{\phi\phi} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \phi \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{G} \end{Bmatrix} \tag{3.27}$$

where $\mathbf{u}$, ϕ, $\mathbf{F}$ and $\mathbf{G}$ are the global vectors of displacement, electric potential, applied force and charge. The element matrices and vectors in Eq.(3.27) are given in previous section.

Different case studies are performed as shown in Fig. 3.2. In all the case studies the piezoelectric control system is modelled by finite element method. The structure is divided into different elements as shown with numbers. After modelling with finite elements mechanical/thermal disturbances are imposed on the structures. The mechanical disturbance are in the form of step or impulse input while the thermal disturbance are of the form of uniform steady temperature increase or heat flux. The best settings of design variables for piezoelectric actuators are found using Taguchi methodology for the best control performance.

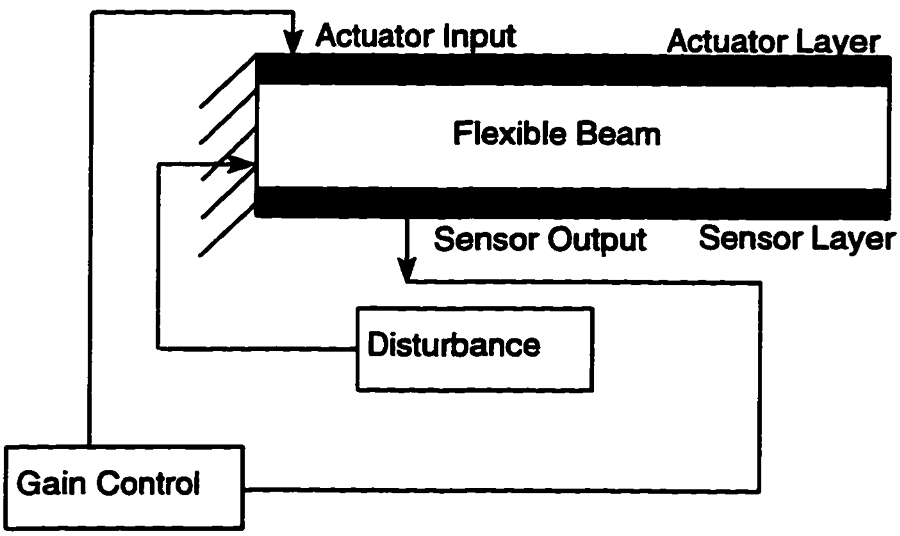

Figure 3.1: Piezoelectric Control System with Sensor and Actuator

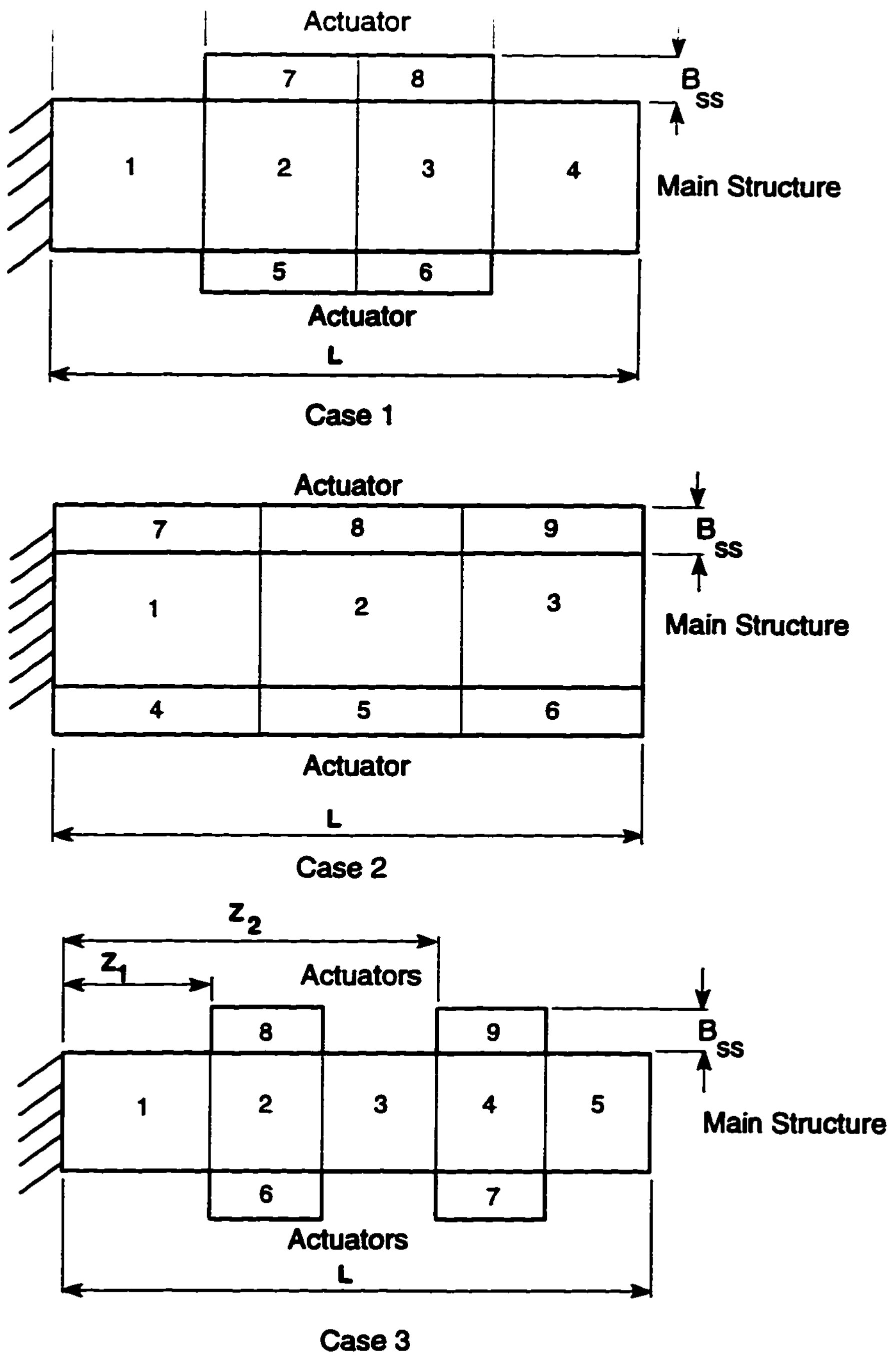

Figure 3.2: Finite Element Model for Different Cases of Piezoelectric Systems

Chapter 4

Robust Design Methodology

4.1 Introduction

Among many, one of the definitions of a robust design methodology can be stated as to retain the performance of a system at least at a prescribed level despite the parameter variablity and disturbances. Robust design is an engineering methodology for improving productivity during research and development so that high-quality products can be produced quickly and at low cost. Robust design offers simultaneous improvement of product quality, performance and cost, and engineering productivity. Its widespread use in industry is having a far-reaching economic impact because this concept can be applied in many engineering activities, including product design and manufacturing process design.

The fundamental principle of robust design is to improve the quality of a product by minimizing the effect of the causes of variation without eliminating the causes. A good amount of time is spent to find different parameters affecting the system performance and the levels of the effects. Once the parameters are known and the effects are put in order with respect to their importance, a sound judgement can be made on the design and manufacture of the system and great savings in cost can be achieved.

4.2 Taguchi Methodology of Robust Design

The foundations of Taguchi methodology were developed in 1950's and 1960's to meet the challenge of producing high-quality products. Since then the methodology has been successfully applied to various fields such as in electronics, automotive products, photography, and many others [29].

Through his research in the 1950's and early 1960's, Taguchi developed the foundations of robust design and validated the basic, underlying philosophies by applying them in the development of many products. The main feature of Taguchi's methodology is parameter design in which the best nominal values for the design variables are selected. Taguchi defines the best values as those values that minimize the transmitted variability resulting from noise factors. This is the reason why Taguchi's methodlogy is referred to as parameter design technique for Robust Design.

All engineering designs involve setting values of a large number of decision variables. Technical experience together with experiments, through prototype hardware models or computer simulations, are needed to come up with the most advantageous decisions about these variables. Studying these variables one at a time or by trial and error is the common approach to the decision process. This leads to either a very long and expensive time span for completing the design or premature termination of the design process and hence the product design is nonoptimal. The robust design proposed by Taguchi uses a mathematical tool called OA's (orthogonal arrays) to study a large number of decision variables with a small number of experiments. The purpose of using OA's is as follows:

- To study the effects of control factors,
- To study the effects of noise factors,
- To evaluate the S/N ratio and
- To determine the best quality characteristic for particular design levels.

The columns of an orthogonal array are pairwise orthogonal i.e for every pair of columns, all combinations of factor level occur an equal number of times. The columns of the OA represent factors to be studied and the rows reperesent individual experiments.

The OA also uses a new measure of quality, called S/N (signal-to-noise ratio) to predict the quality from the customer's perspective, where signal is the desirable part of the objective function and specifies the intended value of the product's response where as noise is attributed as the quality loss to the customer. Noise factors are those factors that cannot be controlled by the designer. Factors whoose settings are difficult or expensive to control are also called noise factors. The noise factors themselves can be divided into three broad classes:

- external - environmental and load factors,
- unit-to-unit variation - manufacturing non uniformity and
- deterioration - wearout, process drift.

In this study, various noise factors in the form of mechanical and thermal disturbances are imposed on the system. The mechanical disturbance is in the form the step or impulse input to the system whereas the thermal disturbance is in the form of uniform steady temperature increase or application of heat flux to one side of the system.

Thus, minimizing the quality loss means maximizing the S/N. Hence, the most economical product and process design from both manufacturing and customer's viewpoints can be accomplished at the smallest, affordable development cost. Taguchi suggested that the steps for a product or a process design are composed of three levels namely, system, parameter and tolerance design. These are explained as follows:

4.2.1 System Design

System design is the phase where new concepts, ideas, methods etc are generated to provide new or improved products. System design is the selection of the major design architecture or process method.

4.2.2 Parameter Design

Parameter design phase is crucial in improving the uniformity of a product and can be done at no cost or even at a saving. The objectives of parameter design experiment are:

- Making products/processes insensitive to manufacturing variations (these include variation in the product parameters from unit to unit due to inherent variation in any manufacturing process).

- Making products/processes insensitive to external factors such as ambient temperature, relative humidity, supply voltage, vibration and so on.

The idea of performing a parameter design experiment/simulation is to minimize the effect of hidden factors termed as noise factors which are hard or expensive to control during normal production conditions. Noise factors are those which cause the performance characteristics of a product to deviate from their nominal values. In order to minimize the effect of these factors, Taguchi recommended the use of control factors/design parameters. For conducting a parameter design experiment/simulation, one first has to construct a design parameter matrix (control array) and then a noise factor matrix (noise array), if any noise factors are considered for the experiment/simulation. The objective of the experiment/simulation is to identify the settings of the design parameters (control factors) which will reduce the effect of noise factors to a minimum. Parameter design improves quality without increasing ultimate maintenance cost. Hence from the above discussion we can say that Robust Design methodology focusses on how to perform parameter design effectively.

4.2.3 Tolerance Design

The tolerance design is implemented to improve quality at minimum cost. It should be used when the sensivity of responses resulting from the parameter design is not satisfactory.

The interaction between control and noise factors is examined to achieve the state of robustness. Taguchi recommends the use of the S/N to evaluate the effect of this interaction with regard to robustness. The S/N gives a measure of the product's functional performance in the presence of noise factors for a given quality characteristic. Quality characteristic refers to the objective function for a given process. S/N for parameter design experiments is further classified into four types in order to improve the performance of a product/process based on the quality loss function. These are as follows :

4.2.4 S/N for smaller-the-better quality characteristic

Examples of this quality characteristic are porosity, shrinkage, level of impurity, vibration, surface roughness of a machined part etc. The quality loss function for smaller-the-better type is given by

$$L(y) = \frac{A_0}{\Delta^2} f_i^2$$

where Δ is the functional limit and A_0 is the loss incurred at the functional limit. The ideal or the target value for this case is given to be zero, the smaller the number of deflections the better is the control performance. Hence, by adopting the quality loss function, we see that the objective function to be maximized is [11, 34]:

$$\text{S/N} = \text{-10 } \log_{10}(\text{mean square deflections})$$

In other words

$$S/N = -10 log_{10} \left[\frac{f_i^2}{n} \right] \tag{4.1}$$

where f_i is the ith individual response value and n is the number of observations. Maximizing S/N leads to minimization of the quality loss due to vibrations or deflections.

Since S/N = -10 $\log_{10} \left[\frac{\mu_i^2}{\sigma^2} \right]$, μ^2/σ^2 is known as the S/N ratio because σ^2 is the effect of noise factors and μ^2 is the desirable part of the objective function. Maximizing (μ^2/σ^2) is equivalent to maximizing S/N which is inturn equivalent to minimizing the quality loss i.e minimizing sensitivity to noise factors.

4.2.5 S/N for larger-the-better quality characteristic

Examples of this quality characteristic are strength of steel, fuel efficiency, life of a battery, hardness of a drill bit etc. The quality loss for this case is given by the following equation

$$L(y) = A_0\Delta^2\left[\frac{1}{f^2}\right]$$

where the target value is ∞. The S/N for larger-the-better quality characteristic is given by [11, 34]:

$$S/N = -10log_{10}\frac{1}{n}\left[\frac{\sum f_i^2}{n}\right] \tag{4.2}$$

4.2.6 S/N for nominal-the-best quality characteristic

Examples of this quality characteristic are dimensions such as diameter, thickness, length, resistance or a resistor and ouput voltage of a power supply. The quality loss for this case is given by

$$L(y) = \frac{A_0}{\Delta^2}(f - m)^2$$

where the target is a prescribed value, namely m. The S/N for nominal-the-best quality characteristic is given by [11, 34]:

$$S/N = 20log_{10}\left[\frac{\bar{f}}{s}\right] \tag{4.3}$$

where $\bar{f}$ and s are the mean response and standard deviation of response values per experimental run/simulation, respectively.

4.3 Taguchi Methodology as Applied to Piezoelectricity

The robust design of a cantilever beam with a pair of piezoelectric actuators is performed so that the system is insensitive to noises which are in the form of mechanical or thermal disturbances. Control and noise factors which affect the control efficiency are identified. Control performance is measured through the vibration simulations of the piezoelectric system. The magnitudes of the oscillations in a given period are summed which is termed as control cost and is signified as the objective function. Also considered as the objective function is the effective damping time of structural vibrations. The smaller the objective function the better is the control performance. The case studies considers factors at three levels and take the levels sufficiently for a part so that a wide region can be covered by the three levels. The steps involved in the robust design are as follows:

- Identify the different parameters and the interactions involved in the process which might affect the objective function as well as the process considerably.

- Select the levels of the parameters depending upon engineering judgement as well as the physics of the problem.
- Select a suitable OA depending upon the number of factors and their levels. For two level design, L_4 or L_8 OA is used and for three level design, L_9 OA is used.
- Depending upon the levels in the OA perform the finite element simulations and obtain the necessary objective function/control efficiency value.
- Having the value of the control efficiency necessitates the evaluation of S/N for smaller-the-better characteristic.
- After the evaluation of the S/N a judgment is made to determine the parameter level best for the process.
- The effect of each factor is confirmed using ANOVA test which aids in reducing the response variability.
- If the objective of the simulation is achieved then a confirmatory run is to be performed; however, if the objective is not achieved, then one should select proper levels and interactions or proper orthogonal array depending upon the circumstances.

4.3.1 L_9 OA as Applied to Case Studies

Table 4.1 shows the Taguchi L_9 array and Fig .4.1 shows the linear graph for Taguchi L_9 array. Linear graph is needed to evaluate the main effects and interactions [29]. The main effects are assigned to points, which designate columns 1 and 2 while columns 3 and 4 deal with the interactions between columns 1 and 2. In this study, while doing preliminary simulations it was found that the thickness (B_{ss}) and the width (W_s) were found to be having predominant effects on the objective function, hence B_{ss} and W_s were assigned to first and second columns respectively. For Case 1, since there were 4 factors, the remaining two factors, namely location of actuators from fixed end (Z) and length of the actuators (d) were assigned to 3rd and 4th columns respectively. For Case 2, nothing was assigned to these columns and these columns were used to study the interaction effects between B_{ss} and W_s, respectively. For Case 3, distance of the first actuator pair from fixed end (Z_1) and distance of the second actuator pair from fixed end (Z_2) were the design variables which were assigned to columns 1 and 2, respectively, while the remaining columns were used to study the interaction effects between Z_1 and Z_2.

4.3.2 Terms and Their Significance in OA and ANOVA Test

Main Factor and Interactions: The main effects of the factors are their individual effects. If the effect of a factor depends on the level of another factor, then the two factors are said to have an interaction, otherwise, they are considered to have no interaction.

Sum of Squares (S_x): The sum of squares is a quantitative measure of the magnitude of its effect due to changes in its level. It is obtained from the following formula for three level design

$$S_x = \left[\frac{f_1^2 + f_2^2 + f_3^3}{n/3}\right] - \left[\frac{T^2}{n}\right]$$

where f_1, f_2, f_3 are the objective function values at first, second and third level respectively. In the above defination, n is the total number of simulations and T is the total of all simulations for a design variable at each variable.

Degrees of Freedom (dF): The number of independent parameters associated with an entity like a matrix experiment, or a factor, or a sum of squares is called its degrees of freedom. A matrix experiment with nine rows has 9 degrees of freedom and so does the grand total sum of squares. The overall mean has one degree of freedom and so does the sum of squares due to mean. Thus, the degrees of freedom associated with the total sum of squares is 9-1 = 8. In general, the degrees of freedom associated with a factor is one less than the number of levels [11].

Pooling: One of the way of estimating the error variation is to use columns which have been assigned factors which are suspected to be significant, but which are shown by the results to be insignificant. In general, if the variation of a factor is less than that of an error column, if it is significantly lower than that of some other columns with factors in them, it can be considered random, and it can be pooled with other insignificant factors and error columns to provide a data base for estimating the random variation of the experiment. The decision of whether or not to pool data from a particular factor can be subjective, i.e for the experiments having no error columns, a judgment is made to pool some of the least significant factors [8]. In the simulations related to this study, all the S_x's were compared with those of the larger of the source variations to the smaller ones. If there was enormous difference between the two then the smaller S_x's were pooled. These factors are designated by a "yes" in the column labeled "Pool" in ANOVA Tables. Their values are then transferred to the columns labeled "dF_e" for degrees of the freedom of the error, and "S_e" for source variation of the error.

Variance of the Source (V_x): The variance of the source, V_x, is the variation of the source corrected for the number of degrees of freedom, according to the formula

$$V_x = \frac{S_x}{dF_x}$$

Pure Variation of the Source ($\mathbf{S'_x}$): The pure variation of the source is calculated only for unpooled factors. It is the variation of a factor with the portion due to error variance removed, and is calculated by the formula

$$S'_x = S_x - (dF \times V_e)$$

F-test: The variance ratio, denoted by F is the ratio of the mean square due to a factor and the error mean square. A large value of F means the effect of that factor is large compared to the error variance. Also, the larger the value of F, the more important that factor is in influencing the process response S/N. In other words, F-test is the statistical test for significance. The F-ratio of the source is calculated for significant or unpooled factors only.

$$F_x = \frac{V_x}{V_e}$$

where is V_x is the variance of source and V_e is the variance of error. **Percent Contribution of the Source to Total Variation ($\rho\%$):** The percent contribution of the source to the total variation is also calculated for unpooled factors. It is calculated with the following formula

$$\rho\% = \left[\frac{S'_x}{S_T}\right] 100\%$$

where S_T is the total sum of squares which is evaluated by summing S_x's of all the factors.

A computer program is developed to determine S/N and variance ratios for important parameters. The flow chart of the program is given as shown in Fig. 4.2.

Table 4.1: Taguchi L_9 Orthogonal Array

Exp No.	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

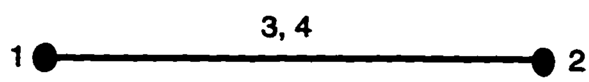

Figure 4.1: Linear Graph for L_9 Orthogonal Array

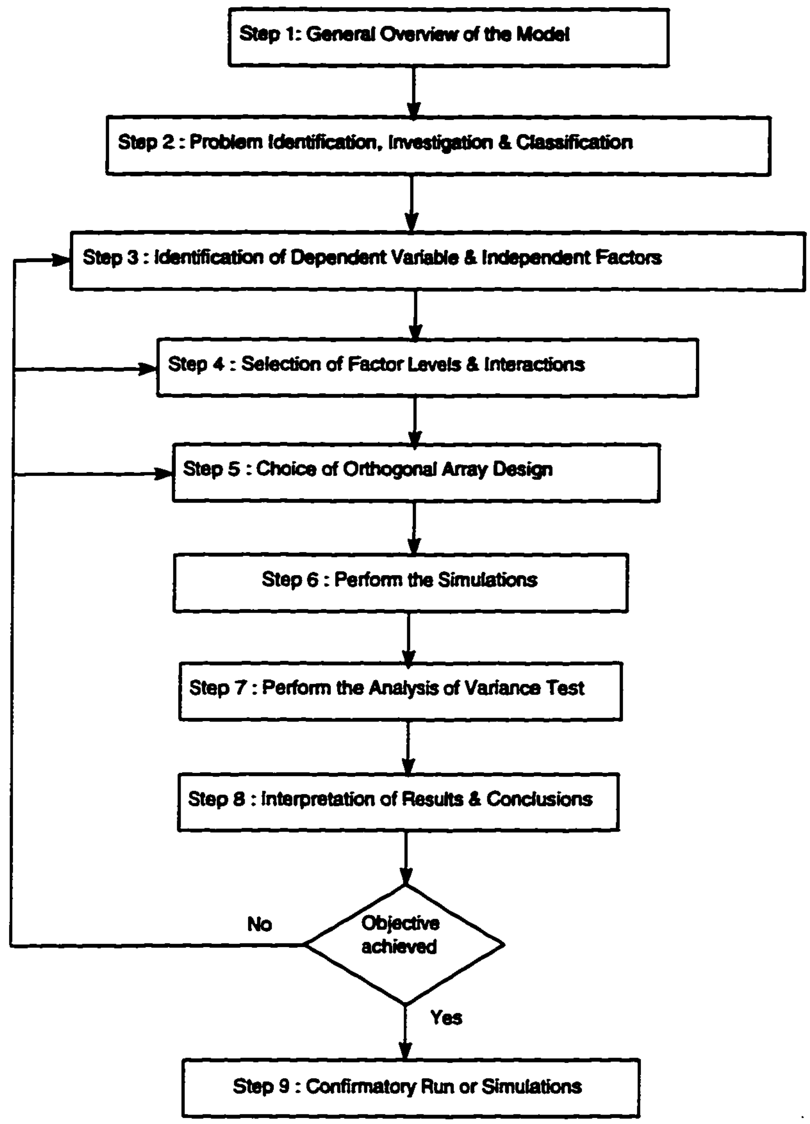

Figure 4.2: Taguchi Methodology of Robust Design

Chapter 5

Case Studies for Piezoelectric Systems

5.1 Preliminary Results

This example [18] is used to check the accuracy of the 2-D finite element model developed for piezoelectric materials. The piezoelectric bimorph beam was clamped at one end and free at the other end as shown in Fig 5.1.

The analytical displacement for the above model is given by [18]

$$u(x) = \frac{3}{2}\left[\frac{e_{31}V}{E(Bss)^2}\right]x^2 \tag{5.1}$$

where e_{31} is the coupling coefficient, V is the applied voltage and Y is the Young's modulus of elasticity.

The finite element model is given by

$$[M_{uu}]\,\ddot{\mathbf{u}} + [K_{uu}]\,\mathbf{u} + [K_{u\phi}]\,\phi = F \tag{5.2}$$

$$[K_{u\phi}]\,\mathbf{u} + [K_{\phi\phi}]\,\phi = G \tag{5.3}$$

The above equations can be combined for static case with no external force to yield the transverse displacement $\mathbf{u}$ as

$$\Big[[K_{uu}] - [K_{u\phi}]\,[K_{\phi\phi}]^{-1}\,[K_{\phi u}]\Big]\,\mathbf{u} = -\,[K_{u\phi}]\,[K_{\phi\phi}]^{-1}\,G \tag{5.4}$$

Simulations were performed first by keeping voltage constant and the displacements were evlauated at each node and in the second case different voltages were applied and the displacements were evaluated. As shown in Figs. 5.2 and 5.3 there is a close agreement of the results between the analytical model and the finite element model which indicates the accuracy of the finite element model.

5.2 Case Studies

Three cantilever beam-like structures (master structures) mounted with piezoelectric actuator pairs are considered in case studies. The master structure and piezoelectric actuator pairs are made up of aluminum and PVDF, respectively. The material properties are listed in Table 5.1. As shown in Fig. 5.4 the master structures are kept same in all the cases, but the configuration, number and location of piezoelectric actuator pairs are varied in each case to observe the effects of design variables on the objective function. The design variables are explicitly stated below for each case. The structures are piezo-controlled using the linear quadratic regulator (LQR) control technique [35]. The controlled structures (closed-loop systems) are simulated for impulsive forces applied at the tip in vertical direction and the vertical displacements (deflections) of the tip are computed. The absolute values of tip deflections in the total simulation time are summed to obtain the first objective function representative of the piezoelectrical control performance for all the cases. The minimum the variation in deflection about its target value the better is the control performance. A second objective function is defined as

$$f = \frac{tr(P)}{tr(Q)} \tag{5.5}$$

which is representative of the effective damping response time [36]. In the above equation tr reperesents the trace which is the sum of the diagnol elements of a matrix. The magnitude of f indicates the effect of control system in reducing vibrations. Effective damping time is composed of control energy and displacements which are represented by P and Q matrices in LQR control technique [36]. In the following tables, the subscript I and II refer to the runs where first and second objective functions are used.

After the design variables are chosen and the objective function is computed, Taguchi methodology is applied to all the cases following the procedure given in the previous Chapter. A constrained problem is also set up and solved for case 3 to check the results with those of Taguchi methodology. The designs at which the SNR of Taguchi methodology attains the highest and lowest values are named as best and worst designs respectively. The closed-loop systems are simulated at these designs for the tip deflections in response to vertical impulsive forces at the tip.

5.3 Case 1

The piezoelectric actuator pairs in Case 1 of Fig. 5.4 covers only certain portions of the top and bottom surfaces of the structure. Thus, in addition to the thickness and width of the actuators (B_{ss} and W_s), the distance of the actuators from the fixed end of the structure (Z) and the length of the actuators (d) are considered as the design variables affecting the control performance. Note that the control performance is evaluated by the two objective functions. The length (L) and height of the structure (h_b) are fixed at L=0.5 m and $h_b = 0.01$ m. The width of the structure and actuators (W_s) is assumed to be varying between 0.01 m and 0.1 m. The thickness of the actuators (B_{ss}) varies between 0.0002 m and 0.0008 m, the location of the actuators varies between 0.05 m and 0.2 m and finally the length of the actuators varies between 0.2 and 0.26 m. Three factor levels are assumed for each design variable as listed in Table 5.2. The major reason in fixing the limits of Z and d as given in the table is to keep the sizes of finite elements in reasonable ranges.

The L_9 OA, objective function and SNR values are given in Table 5.3. The design variables given in Tables 5.2 have effects of different magnitudes on both objective functions as shown in Tables 5.3 through 5.5 and Figs. 5.5 and 5.6. The results based on Taguchi methodology within the levels of design variables reveal that the thickness of the actuators (B_{ss}) has the biggest effect on the control performance. The higher magnitudes of B_{ss} result in better control performance. A similar trend is observed for the width of the actuators (W_s) with lesser magnitude relative to the first objective function. The distance of the actuators from the fixed end (Z) also affects the control performance, however the effect is the reversal of the effects of B_{ss} and W_s. In other words, the higher magnitudes of B_{ss} and W_s, and the lower magnitude of Z result in better control performance characteristics. The effect of the actuator length (d) appears to be negligible within the bounds considered in this case. The best design with respect to the first objective function is noted from Table 5.3 to be at : B_{ss} = 0.0008 m, W_s = 0.05 m, d = 0.26 m and Z = 0.05 m, and the worst design is observed at : B_{ss} = 0.0002 m, W_s = 0.01 m, d = 0.2 m and Z = 0.05 m. The best design with respect to the second objective function is noted from Table 5.3 to be at : B_{ss} = 0.0008 m, W_s = 0.01 m, d = 0.23 m and Z = 0.2 m, and the worst design is observed at : B_{ss} = 0.0002 m, W_s = 0.1 m, d = 0.26 m and Z = 0.2 m. Tip deflection plots of the closed-loop system are shown in Fig. 5.7 for the best and worst designs with respect to the first objective function. The plots clearly indicate that the system at the best design performs much better.

There is a variation in the optimum levels from first objective function to second objective function because in the first objective function there is a minimization of only the deflections from the target value, whereas in the second objective function minimization of both the deflections and control energy is done. Simulation results indicate that only the thickness has predominant effect for first objective function whereas for second objective function there is some minimal effect due to width of the actuators also. In other words for first objective function higher dimensions of B_{ss}, W_s, d and lower dimensions of z give better control performance. For second objective function higher dimensions of B_{ss}, z, d and lower dimensions of W_s give better control performance.

5.4 Case 2

The master structure with a pair of piezoelectric actuators bonded to the whole top and bottom surfaces is shown in Case 2 of Fig. 5.4. The length and height of the structure, and the variance of the width of the structure and actuators are same as in the previous case, but the actuators which have only covered part of the structure now cover the whole length of the structure. The length of the actuators is same as that of the structure and the thickness of the actuators (B_{ss}) varies between 0.0002 m and 0.0008 m. Hence the width and thickness of the piezoelectric actuator pairs (W_s and B_{ss}) are taken as the design variables affecting the control performance.

The levels of design variables are listed in Table 5.6.

The L_9 OA, values of the objective functions and SNR are given in Table 5.7. The first two columns of the OA reflect the variation in the thickness and width of the actuators while the remaining two columns show the effect of the interaction between the thickness and the width of the actuators. The effects of different parameters (design variables and interactions together) are shown in Tables 5.7 through 5.9, and Figs. 5.8 and 5.9. It can be seen easily that the thickness of the actuators (B_{ss}) has a predominant effect on the control performance within the specified range. To a lesser degree, the width of the actuators (W_s) has the similar effect on the control performance of the system with respect to the first objective function. In general, the larger the thickness of the actuators, the better is the control performance of the system. The best design relative to first objective function is noted at the upper bounds of design variables, $B_{ss} = 0.0008$ m and $W_s = 0.1$ m, and the worst design at the lower bounds, $B_{ss} = 0.0002$ m and $W_s = 0.01$ m. The best design relative to second objective function is noted at the upper bounds of design variables, $B_{ss} = 0.0008$ m and $W_s = 0.01$ m, and the worst design at the lower bounds, $B_{ss} = 0.0002$ m and $W_s = 0.1$ m. The superiority of the best design over the worst design with respect to the first objective function is evident in the closed-loop system responses as shown in Fig. 5.10.

In this case also for first objective function higher dimensions of B_{ss} and W_s give better control performance whereas for second objective function higher dimensions of B_{ss} and lower dimensions of W_s give better control performance.

5.5 Case 3

The piezoelectric control system in this case consists of the main structure as before and two piezoelectric actuator pairs as shown in Case 3 of Fig. 5.4. The length, height and width of the structure (L, h_b and W_s) are fixed at 0.5 m, 0.01 m and 0.01 m, and those of the actuators are held constant at $\frac{L}{4}$ = 0.125 m, 0.0005 m and 0.01 m, respectively. The distances of the actuators pairs from the fixed end (Z_1 and Z_2) are taken as design variables. The design variables assume three levels as listed in Table 5.10. The levels are so chosen to ensure reasonable finite element sizes.

The L_9 OA, objective function and SNR values are listed in Table 5.11, and the effects of parameters are shown in Tables 5.11 through 5.13, and Figs. 5.11 and 5.12. It is apparent from the listed and plotted data that the distance of the first actuator from the fixed end (Z_1) has a major effect on the control performance. Overall, the actuators closer to the fixed end function better than those further away from the fixed end for the first objective function whereas for the second objective function it is observed that for obtaining best control performance the first actuator should be away from the fixed end while the second actuator must be close to the fixed end. The best and worst design variable settings with respect to the first objective function are noted at : $Z_1 = 0.005$ m and $Z_2 = 0.32$ m, and $Z_1 = 0.1$ m and $Z_2 = 0.23$ m, respectively. The best and worst design variable settings with respect to the second objective function are noted at : $Z_1 = 0.1$ m and $Z_2 = 0.23$ m, and $Z_1 = 0.05$ m and $Z_2 = 0.32$ m, respectively. The closed-loop system response plots shown in Fig. 5.13 clearly indicate the improvement on the response when the first actuator is placed near the fixed end. Response between best and worse design does not show much variation because only the effect of location of actuators on the control performance is studied for given dimensions of the actuators.

5.6 Optimization Results

A constrained optimzation problem is formulated for Case 3 using the first objective function and design variables. The constraints are imposed to ensure the finite elements of reasonable dimensions. The problem is solved by an optimization algorithm based on sequential quadratic programming [37].

The initial and final values of design variables and objective function are listed in Table 5.14. It is observed that a great reduction in objective function value is achieved at the final design which corresponds to the closest location of the first actuator to the fixed end within the feasible region. This trend is in agreement with the best design found by Taguchi methodology for the first actuator pair with respect to the first objective function and similar conclusions were made in some references using other techniques [22, 38, 39]

Table 5.1: Properties of Materials

PVDF	300 K	325 K
$c_{11}(Pa)$	3.8×10^9	2.6×10^9
$e_{31}(C/m^2)$	0.046	0.047
$P_3(C/m^2K)$	4×10^{-5}	4.5×10^{-5}
$K_{11}(W/mK)$	0.52	0.52
$K_{33}(W/mK)$	0.12	0.12
$\epsilon_{11}(F/m)$	1.026×10^{-10}	1.026×10^{-10}
$\epsilon_{33}(F/m)$	1.026×10^{-10}	1.026×10^{-10}
$\alpha(1/K)$	1.5×10^{-4}	1.75×10^{-4}
$\rho(kg/m^3)$	1800	1800
$c_p(F)$	3.8×10^{-6}	3.8×10^{-6}
Beam		
$c_{11}(Pa)$	7.3×10^{10}	
$\alpha(1/K)$	2.4×10^{-5}	
$K_{11}(W/mK)$	236	
$\rho(kg/m^3)$	2750	

Table 5.2: Levels of Design Variables for Case 1

Bss (m)	Ws (m)	Z (m)	d (m)
0.0002	0.01	0.05	0.2
0.0005	0.05	0.12	0.23
0.0008	0.1	0.2	0.26

Table 5.3: L_9 Orthogonal Array, Objective Function and SNR Values for Case 1

Exp No.	Bss	Ws	Z	d	f_I	SNR_I	f_{II}	SNR_{II}
1	1	1	1	1	0.1064	19.46	4.53E+11	-233.13
2	1	2	2	2	0.0993	20.06	3.44E+12	-250.72
3	1	3	3	3	0.0705	23.03	9.86E+12	-259.88
4	2	1	2	3	0.0538	25.38	1.42E+12	-243.03
5	2	2	3	1	0.0649	23.75	2.26E+12	-247.09
6	2	3	1	2	0.0369	28.65	5.89E+12	-255.41
7	3	1	3	2	0.0465	26.65	1.80E+10	-205.09
8	3	2	1	3	0.026	31.70	1.30E+11	-222.28
9	3	3	2	1	0.0295	30.60	1.22E+11	-221.71

Table 5.4: ANOVA for Case 1 using First Objective Function (f_I)

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S_x'	$\rho\%$
Bss_1	62.55	2	117.05	no			58.52		10.35	105.75	70
Bss_2	77.79										
Bss_3	88.95										
Ws_1	71.49	2	19.87	yes	2	19.87		9.93			
Ws_2	75.51										
Ws_3	82.29										
Z_1	79.82	2	6.85	yes	2	6.85		3.42			
Z_2	76.04										
Z_3	73.44										
d_1	73.81	2	7.18	yes	2	7.18		3.59			
d_2	75.37										
d_3	80.12										
error											
e_{Total}			0		6	33.91		5.65			
Total			150.97								

Table 5.5: ANOVA for Case 1 using Second Objective Function (f_{II})

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S_x'	$\rho\%$
Bss_1	-743.72	2	2029.46	no			1014.73		42.60	1981.82	74
Bss_2	-745.53										
Bss_3	-649.07										
Ws_1	-681.24	2	544.82	no			272.41		11.43	497.18	19
Ws_2	-720.09										
Ws_3	-736.99										
Z_1	-710.82	2	3.85	yes	2	3.85		1.92			
Z_2	-715.46										
Z_3	-712.05										
d_1	-701.93	2	91.41	yes	2	91.42		45.71			
d_2	-711.21										
d_3	-725.19										
error											
e_{Total}			0		4	95.27		23.82			
Total			2669.51								

Table 5.6: Levels of Design Variables for Case 2

Bss (m)	Ws (m)
0.0002	0.01
0.0005	0.05
0.0008	0.1

Table 5.7: L_9 Orthogonal Array, Objective Function and SNR Values for Case 2

Exp No.	Bss	Ws	$Bss \times Ws$	$Bss \times Ws$	f_I	SNR_I	f_{II}	SNR_{II}
1	1	1	1	1	0.724	2.80	5.59E+12	-254.95
2	1	2	2	2	0.707	3.01	8.53E+12	-258.62
3	1	3	3	3	0.487	6.24	1.07E+13	-260.55
4	2	1	2	3	0.284	10.93	3.39E+11	-230.61
5	2	2	3	1	0.279	11.08	4.63E+11	-233.32
6	2	3	1	2	0.266	11.50	6.25E+11	-235.92
7	3	1	3	2	0.177	15.04	7.48E+10	-217.48
8	3	2	1	3	0.173	15.23	1.10E+11	-220.80
9	3	3	2	1	0.17	15.391	1.64E+11	-224.29

Table 5.8: ANOVA for Case 2 using First Objective Function (f_l)

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S_x'	ρ%
Bss_1	12.06	2	193.02	no			96.51		75.22	190.45	95
Bss_2	33.52										
Bss_3	45.67										
Ws_1	28.77	2	3.75	yes	2	3.75		1.87			
Ws_2	29.33										
Ws_3	33.14										
$(Bss \times Ws)_1$	29.54	2	1.92	yes	2	1.92		0.96			
$(Bss \times Ws)_2$	29.33										
$(Bss \times Ws)_3$	32.37										
$(Bss \times Ws)_1$	29.28	2	2.01	yes	2	2.01		1.00			
$(Bss \times Ws)_2$	29.55										
$(Bss \times Ws)_3$	32.42										
error											
e_{Total}			0		6	7.69		1.28			
Total			200.72								

Table 5.9: ANOVA for Case 2 using Second Objective Function (f_{II})

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S'_x	$\rho\%$
Bss_1	-774.12	2	2149.83	no			1074.91		120.58	2131.99	97
Bss_2	-699.84										
Bss_3	-662.57										
Ws_1	-703.03	2	52.491	yes	2	52.49		26.25			
Ws_2	-712.74										
Ws_3	-720.75										
$(Bss \times Ws)_1$	-711.66	2	0.92	yes	2	0.92		0.46			
$(Bss \times Ws)_2$	-713.52										
$(Bss \times Ws)_3$	-711.34										
$(Bss \times Ws)_1$	-712.55	2	0.07	yes	2	0.07		0.03			
$(Bss \times Ws)_2$	-712.01										
$(Bss \times Ws)_3$	-711.96										
error											
e_{Total}			0		6	53.48		8.91			
Total			2203.31								

Table 5.10: Levels of Design Variables for Case 3

$Z1$ (m)	$Z2$ (m)
0.005	0.23
0.05	0.28
0.1	0.32

Table 5.11: L_9 Orthogonal Array, Objective Function and SNR Values for Case 3

Exp No.	$Z1$	$Z2$	$Z1 \times Z2$	$Z1 \times Z2$	f_I	SNR_I	f_{II}	SNR_{II}
1	1	1	1	1	0.1238	18.14	1.52E+10	-203.64
2	1	2	2	2	0.1125	18.97	2.298E+10	-207.21
3	1	3	3	3	0.1032	19.72	1.72E+10	-204.68
4	2	1	2	3	0.1421	16.94	2.04E+10	-206.20
5	2	2	3	1	0.1318	17.6	2.08E+10	-206.38
6	2	3	1	2	0.1212	18.32	3.17E+10	-210.03
7	3	1	3	2	0.1659	15.60	1.21E+10	-201.67
8	3	2	1	3	0.1572	16.07	1.83E+10	-205.24
9	3	3	2	1	0.1473	16.63	1.64E+10	-204.31

Table 5.12: ANOVA for Case 3 using First Objective Function (f_l)

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S_x'	$\rho\%$
$(Z1)_1$	56.84	2	12.17	no			6.08		303.91	12.13	81
$(Z1)_2$	52.87										
$(Z1)_3$	48.31										
$(Z2)_1$	50.69	2	2.66	no			1.33		66.44	2.62	18
$(Z2)_2$	52.65										
$(Z2)_3$	54.69										
$(Z1 \times Z2)_1$	52.55	2	0.03	yes	2	0.03		0.02			
$(Z1 \times Z2)_2$	52.56										
$(Z1 \times Z2)_3$	52.93										
$(Z1 \times Z2)_1$	52.38	2	0.05	yes	2	0.05		0.02			
$(Z1 \times Z2)_2$	52.91										
$(Z1 \times Z2)_3$	52.74										
error											
e_{Total}			0		4	0.08		0.02			
Total			14.91								

Table 5.13: ANOVA for Case 3 using Second Objective Function (f_{II})

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S'_x	$\rho\%$
$(Z1)_1$	-615.54	2	22.03	no			11.01		4.11	16.68	37
$(Z1)_2$	-622.61										
$(Z1)_3$	-611.22										
$(Z2)_1$	-611.51	2	12.23	no			6.11		2.28	6.88	15
$(Z2)_2$	-618.83										
$(Z2)_3$	-619.03										
$(Z1 \times Z2)_1$	-618.91	2	7.16	yes	2	7.16		3.58			
$(Z1 \times Z2)_2$	-617.73										
$(Z1 \times Z2)_3$	-612.73										
$(Z1 \times Z2)_1$	-614.34	2	3.54	yes	2	3.54		1.76			
$(Z1 \times Z2)_2$	-618.91										
$(Z1 \times Z2)_3$	-616.13										
error											
e_{Total}			0		4	10.69		2.67			
Total			44.96								

Table 5.14: Optimization Results for Case 3

	Design Variables (Z_1, Z_2) (m)	Objective Function (f_l) (m)
Initial Design	(0.15, 0.37)	0.1656
Final Design	(0.0055, 0.3784)	0.09
Taguchi Design	(0.005, 0.32)	0.1032

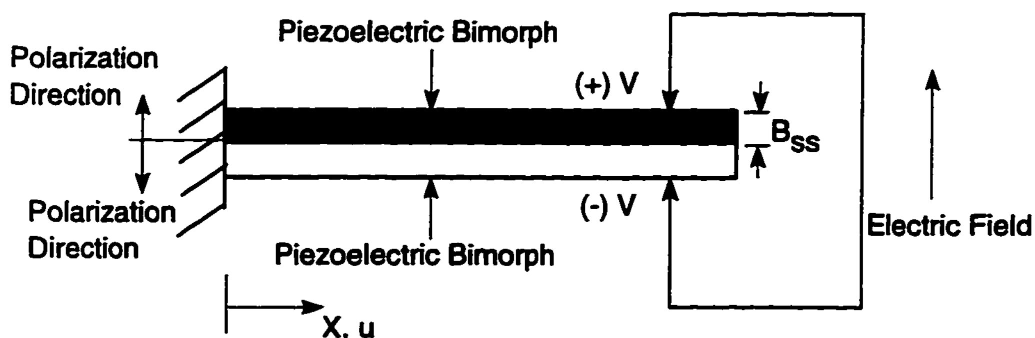

Figure 5.1: Layout of Piezoelectric Bimorph Beam

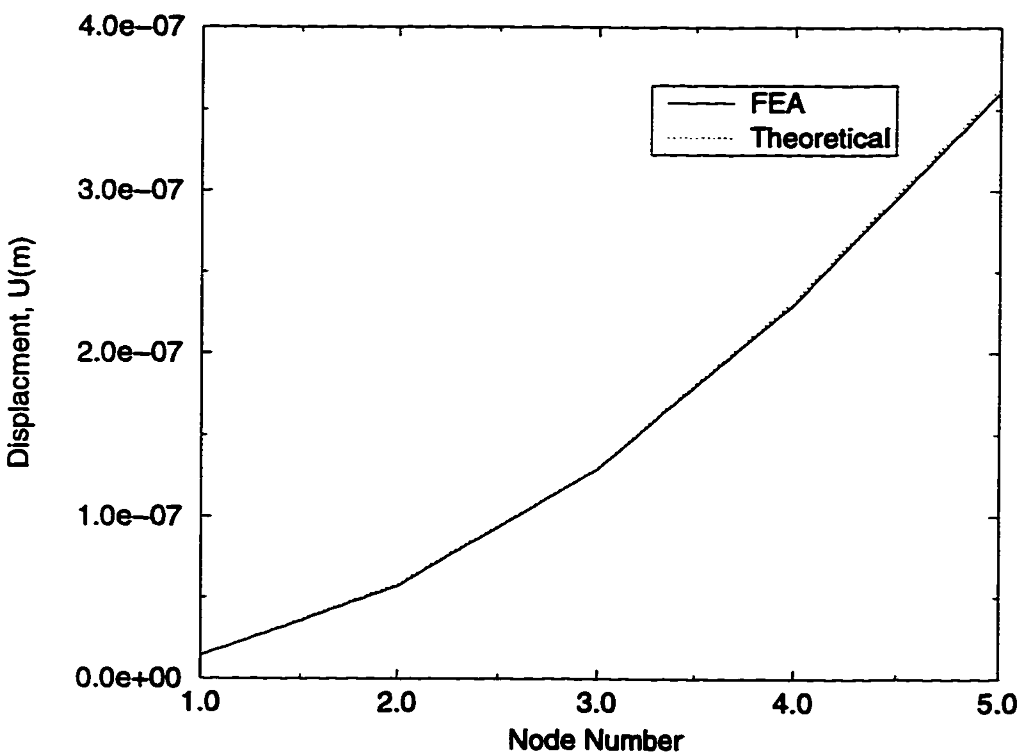

Figure 5.2: Confirmation of Results of Finite Element Analysis

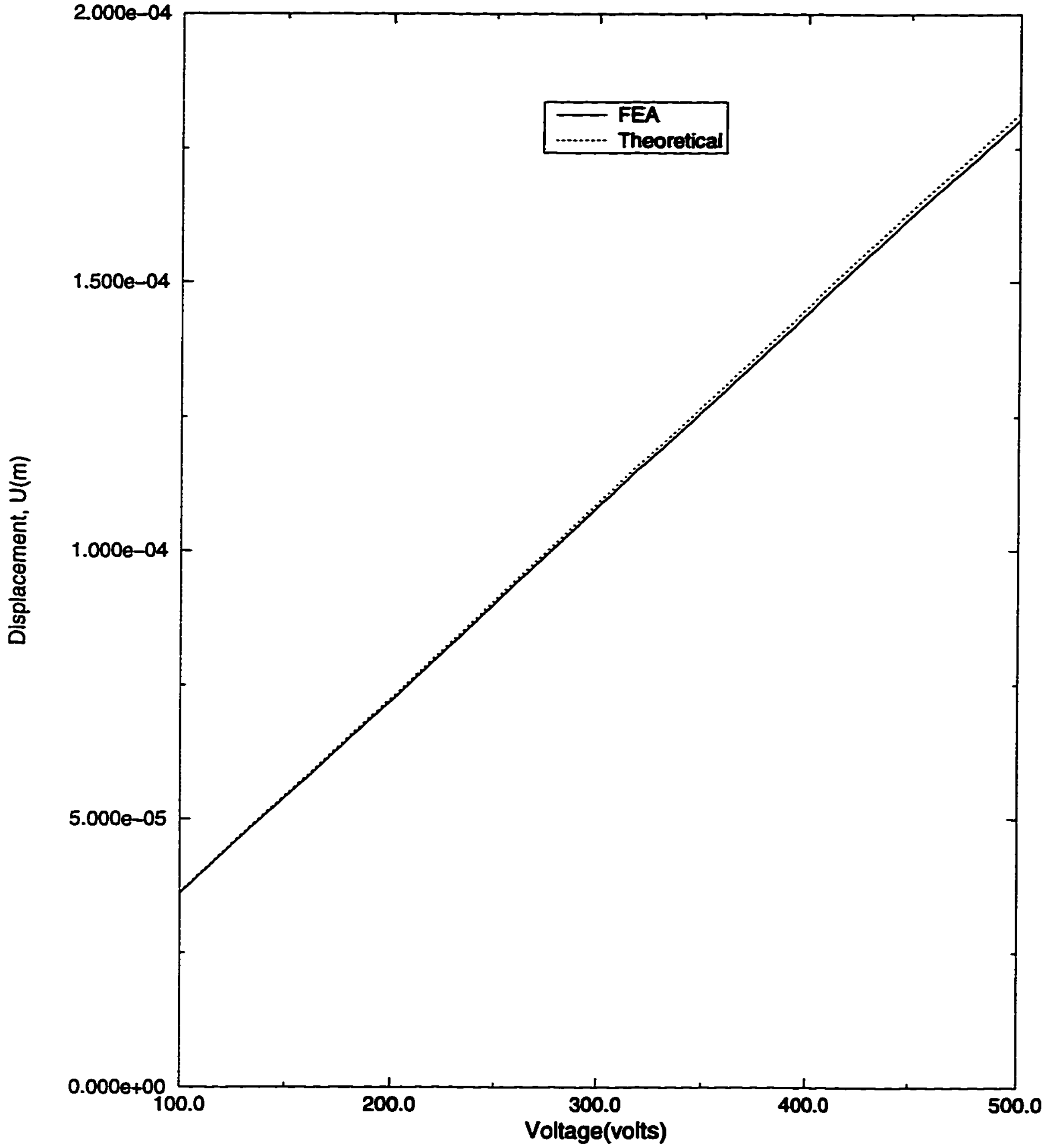

Figure 5.3: Confirmation of Results of Finite Element Analysis

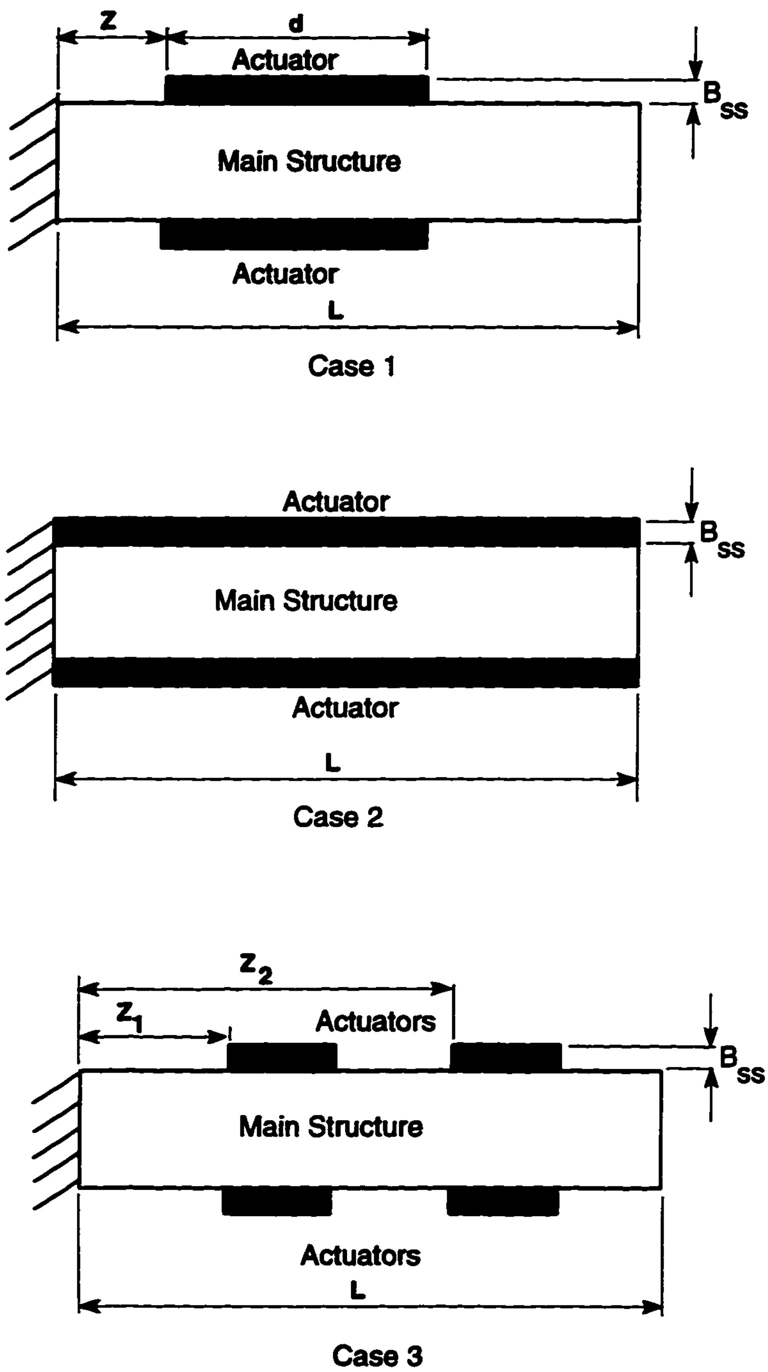

Figure 5.4: Piezoelectrical Control Systems for Different Cases

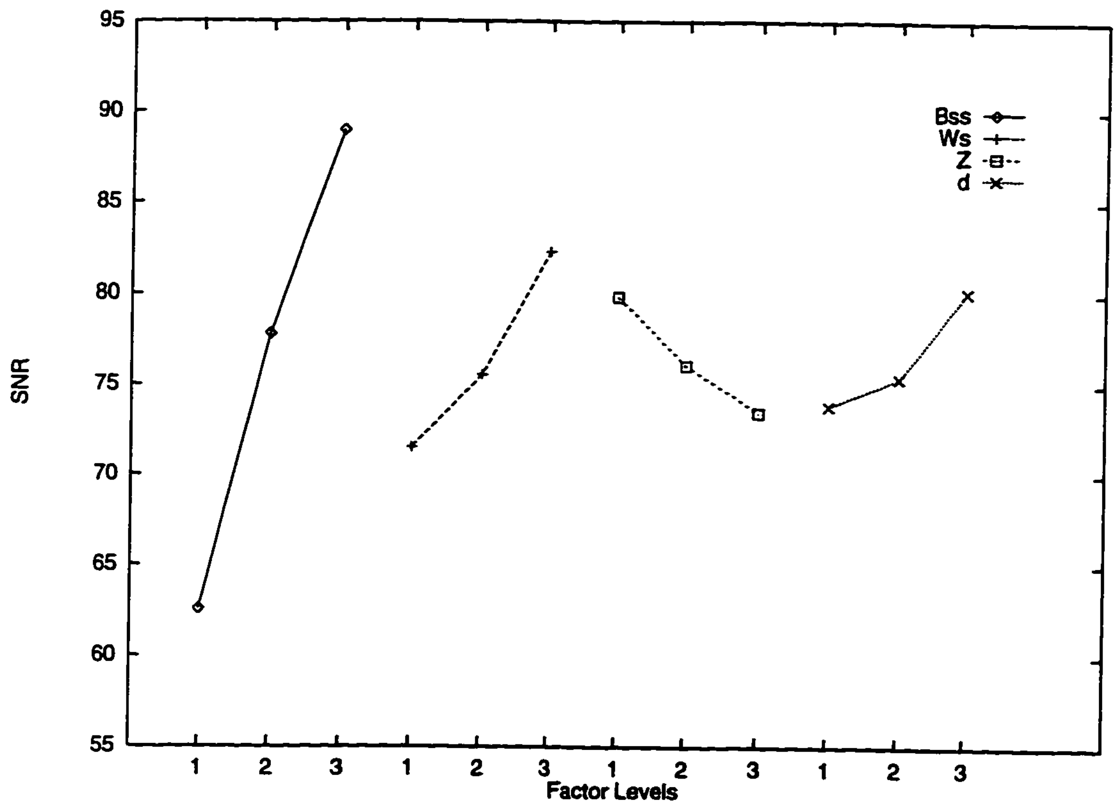

Figure 5.5: Effect of Each Design Variable on First Objective Function for Case 1

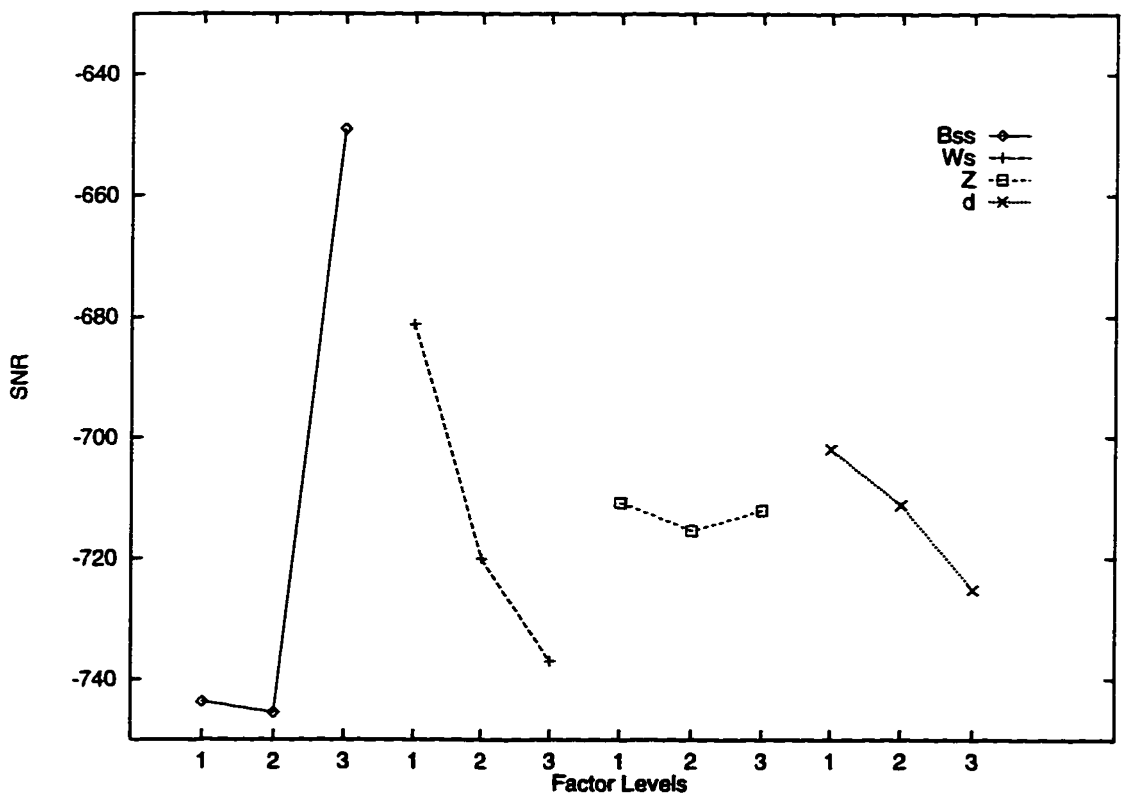

Figure 5.6: Effect of Each Design Variable on Second Objective Function for Case 1

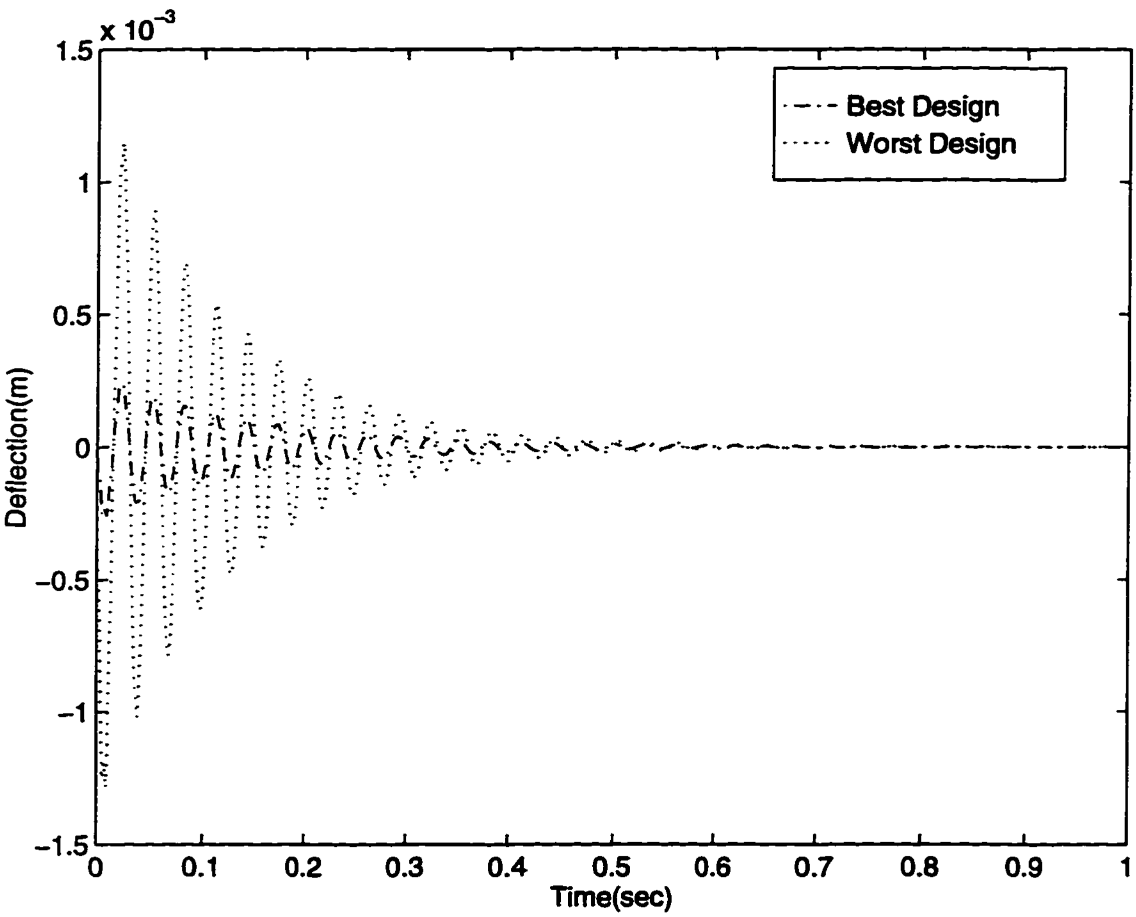

Figure 5.7: Tip Deflection of Closed-Loop System for Case 1 with First Objective Function

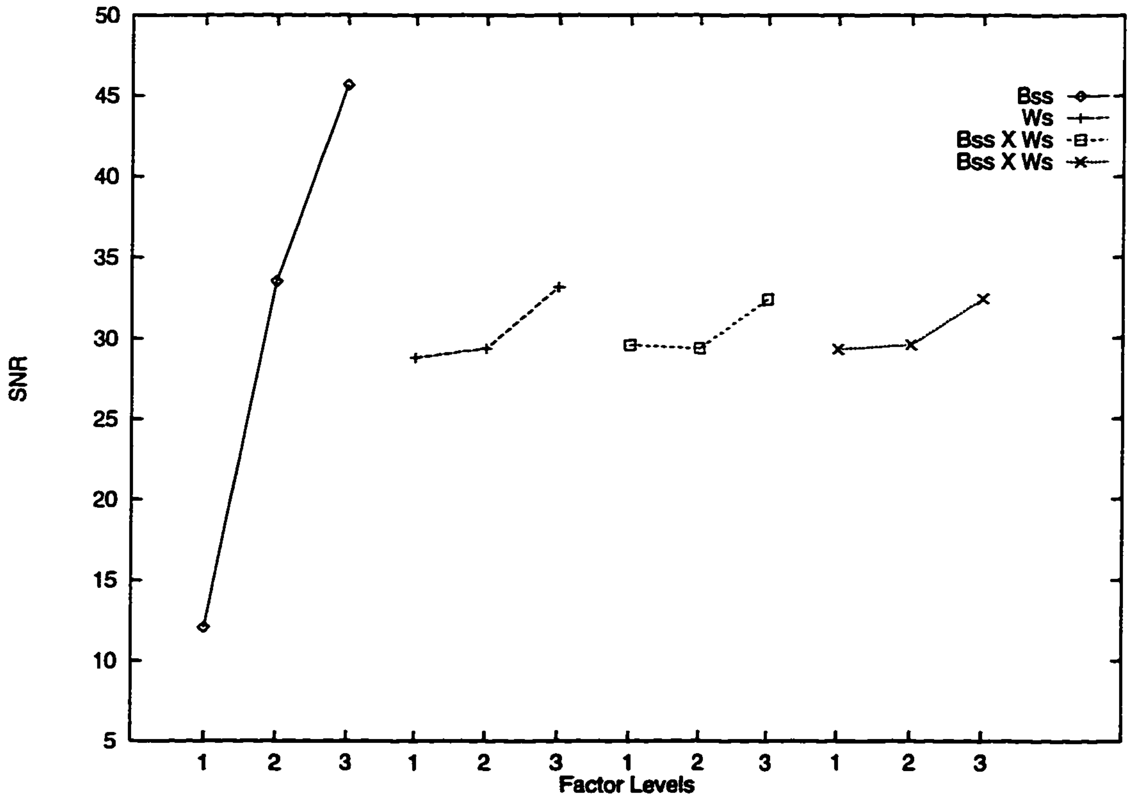

Figure 5.8: Effect of Each Factor on First Objective Function for Case 2

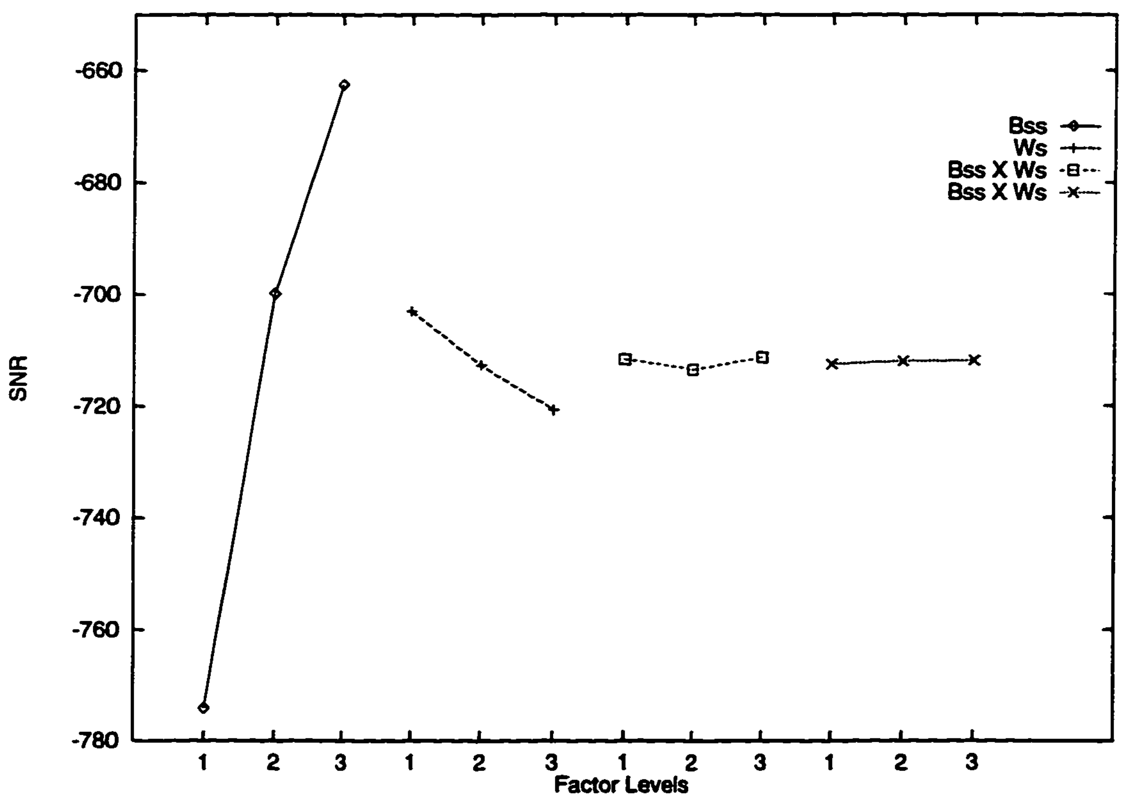

Figure 5.9: Effect of Each Factor on Second Objective Function for Case 2

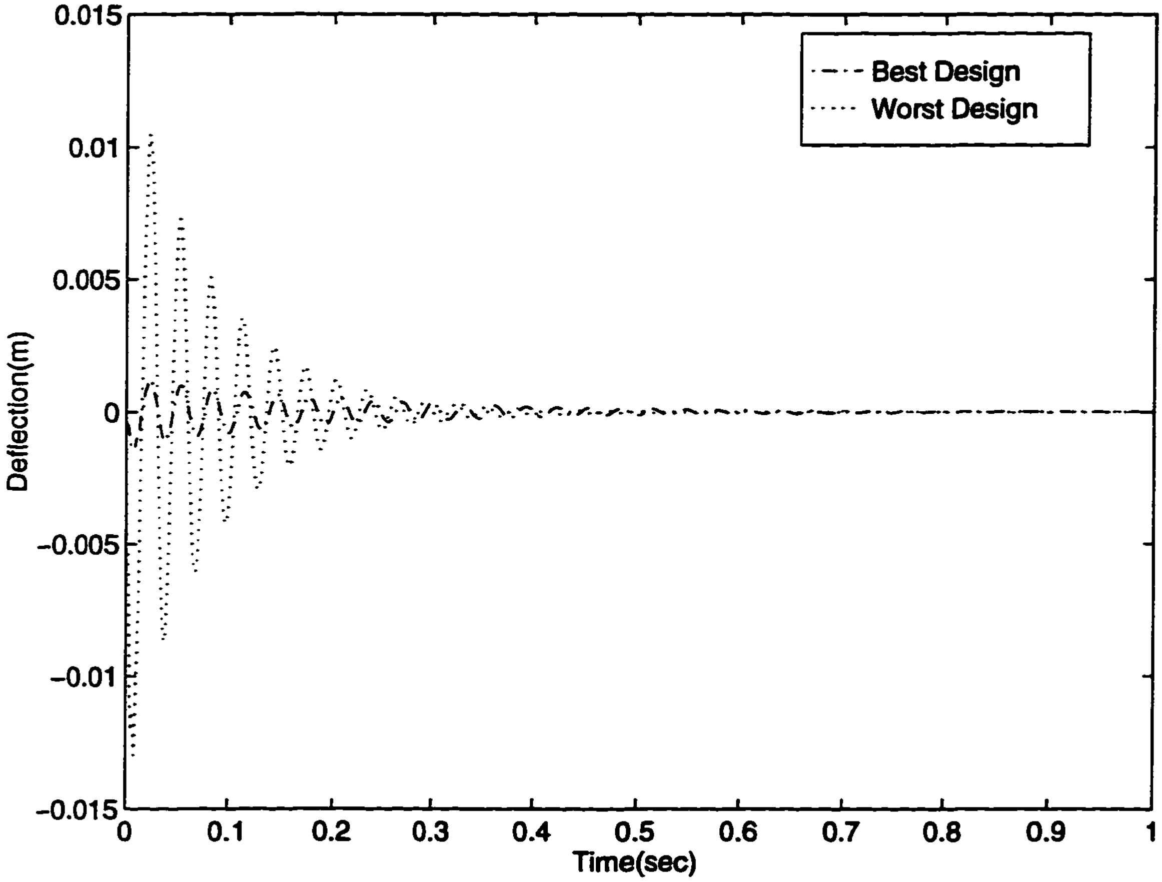

Figure 5.10: Tip Deflection of Closed-Loop System for Case 2 with First Objective Function

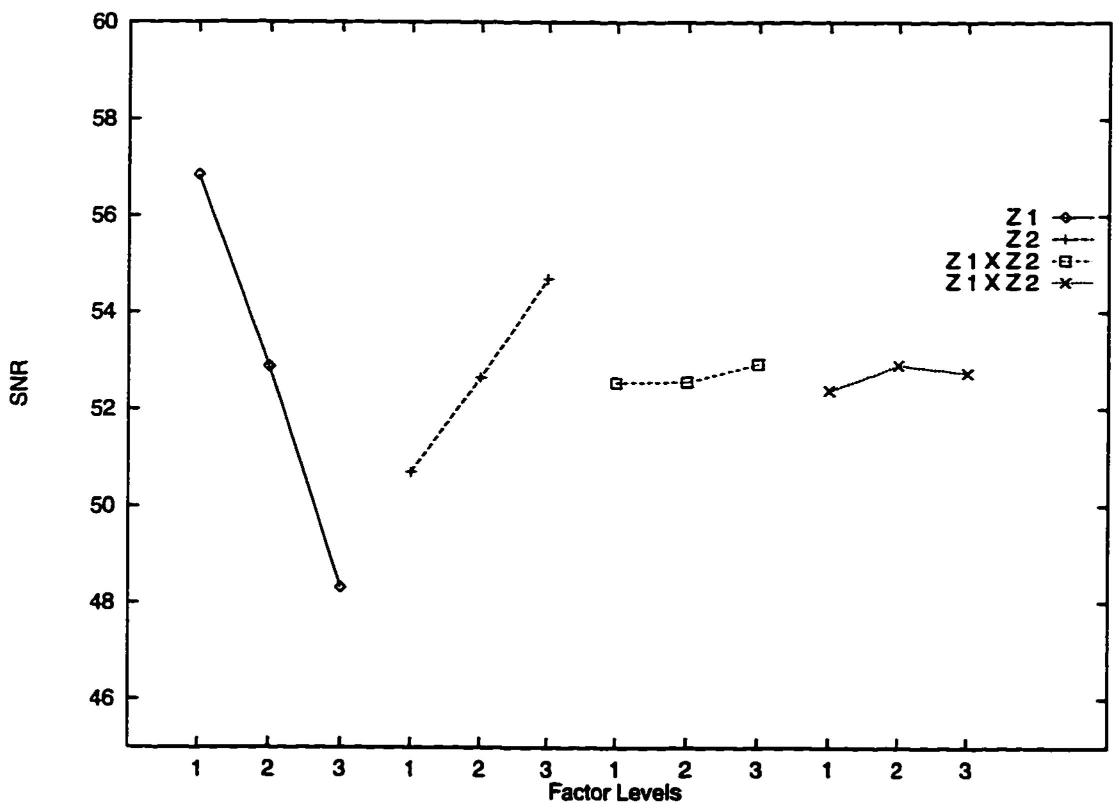

Figure 5.11: Effect of Each Factor on First Objective Function for Case 3

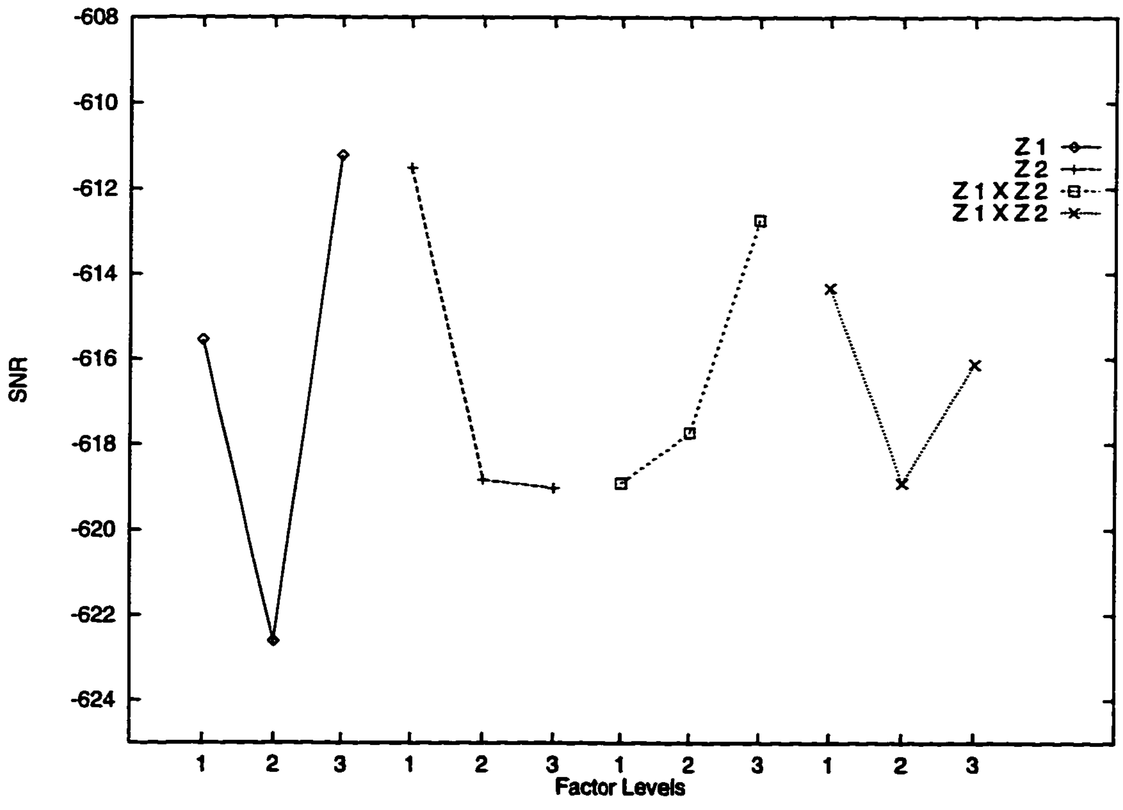

Figure 5.12: Effect of Each Factor on Second Objective Function for Case 3

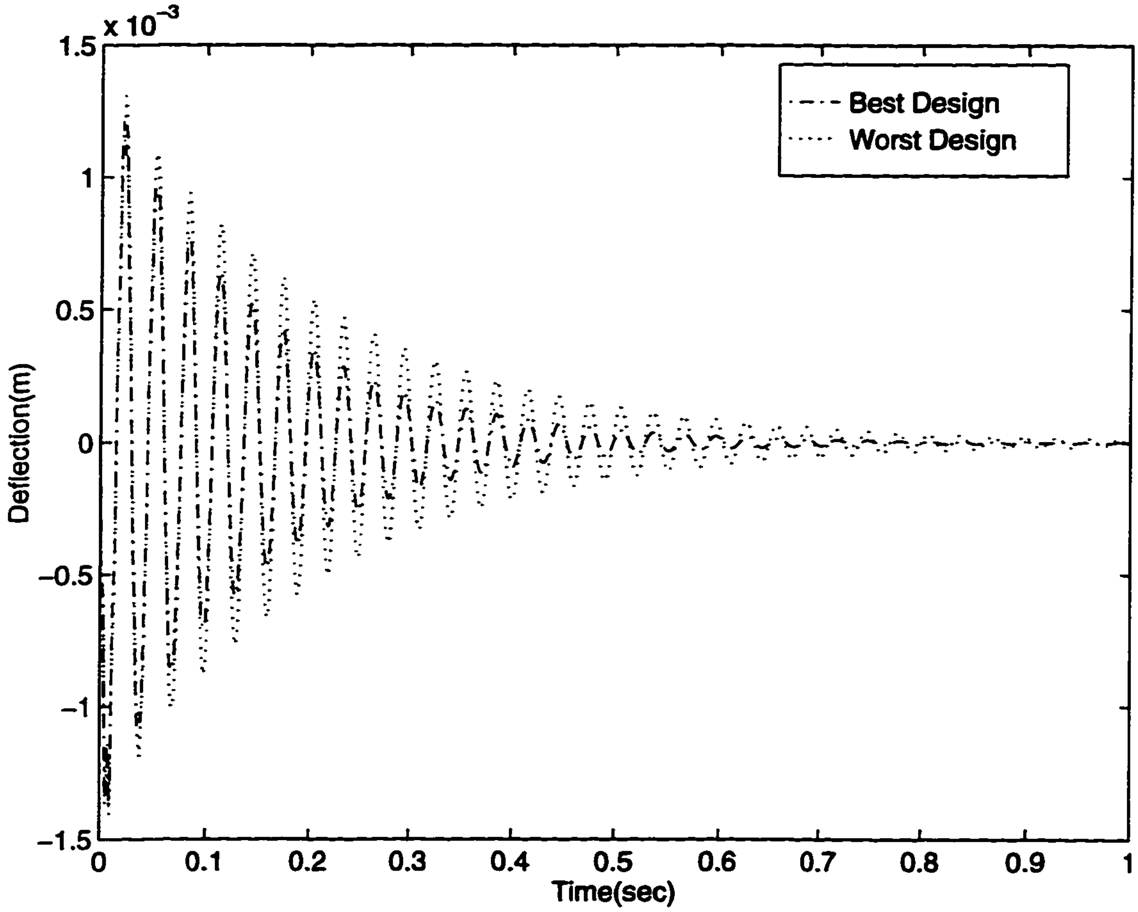

Figure 5.13: Tip Deflection of Closed-Loop System for Case 3 with First Objective Function

Chapter 6

Case Studies for Thermopiezoelectric Systems

Two cantilever beam-like structures (master structures) mounted with piezoelectric actuator pairs are considered in case studies. The two different configurations are the Case 1 and Case 2 of Fig. 6.1. The configurations of the master structure and piezoelectric actuator pairs are same as before for both the cases. Two types of steady-state thermal fields are imposed on the systems. A uniform temperature increase of 25 K for all the systems is first assumed. Then, a heat flux of 500 W/m^2 directed upward is imposed on the bottom side of the systems as shown by arrows in Fig. 6.1. Since the systems are much larger in size in longitudinal directions, the temperature variations due to the heat flux take place mostly in cross-sectional plane. Hence four distinct temperature increases are noted. These are θ_1, θ_2, θ_3 and

θ_4 which respectively represent the temperature increases along the bottom and top sides of the beam, and along the bottom and top sides of the bottom and top actuators. The temperature increases with the variation in dimensions are given in Table 6.1. The controlled structures (closed-loop systems) are simulated for step forces applied at the tip in vertical direction and the vertical displacements (deflections) of the tip are computed. The absolute values of tip deflections in the total simulation time are summed to obtain the first objective function of the thermopiezoelectrical systems for both the cases. The second objective function is defined same as that in the previous Chapter. In the tables which follow, the subscripts 1 and 2 refer to the simulations where thermal fields of the first and second type are imposed.

After the design variables are chosen and the objective functions are computed, Taguchi methodology is applied as before to both the cases following the procedure given in the previous Chapter. As before, the designs at which the SNR of Taguchi methodology attains the highest and lowest values are named as best and worst design, respectively. The closed-loop systems are simulated at these designs for the tip deflections in response to vertical step forces at the tip.

6.1 Case 1

The piezoelectric actuator pairs in Case 1 of Fig. 6.1 covers only certain portions of the top and bottom surfaces of the structure. Thus the design variables affecting the control performance, which is measured by objective functions, are thickness and width of the actuators (B_{ss} and W_s), the distance of the actuators from the fixed end of the structure (Z) and the length of the actuators (d). The configuration of the structure is as before in the earlier chapter and three levels are assumed for each design variable as listed in Table 6.2.

First, Taguchi methodology is applied to the system which is subjected to a thermal disturbance of 25 K temperature increase and is acted upon a unit vertical step force at the tip. The L_9 OA, objective function and SNR values are given in Table 6.3. The design variables given in Tables 6.2 have considerable effects on the objective functions as shown in Tables 6.3 through 6.5 and Figs. 6.2 and 6.3. The results based on Taguchi methodology within the levels of design variables reveal that the width of the actuators (W_s) has the biggest effect on the control performance with respect to the first objective function and the thickness of the actuators (B_{ss}) is the dominant factor with regard to the second objective function. The highest magnitudes of W_s and B_{ss} result in better control performance characteristics for the first objective function whereas for the second objective function highest magnitude of B_{ss} and lower magnitude of W_s result in better control performance. The effects of the actuator distance (Z) and actuator length (d) appear to be negligible within the bounds considered. The best design with respect to the first objective function is noted from Table 6.3 to be at : B_{ss} = 0.0008 m, W_s = 0.1 m, Z = 0.12 m and d = 0.2 m and the worst design is observed at : B_{ss} = 0.0002 m, W_s = 0.01 m, Z =

0.05 m and d = 0.2 m or at B_{ss} = 0.0005 m, W_s = 0.01 m, Z = 0.12 m and d = 0.26 m. The best design with respect to the second objective function is noted from Table 6.3 to be at : B_{ss} = 0.0008 m, W_s = 0.01 m, Z = 0.2 m and d = 0.23 m and the worst design is observed at : B_{ss} = 0.0002 m, W_s = 0.1 m, Z = 0.2 m and d = 0.26 m. Tip deflection plots of the closed-loop system are shown in Fig. 6.4 for the best and worst designs with respect to the first objective function. The plots clearly indicate that the system at the best design performs much better.

Taguchi methodology is further applied to the system with the second type of thermal field while the mechanical disturbance is kept the same. The L_9 OA with the same design variables and levels are considered. SNR values are given in Table 6.3. The results based on Taguchi methodology reveal that the width of the actuators (W_s) has the most predominant effect with respect to the first objective function and the thickness of the actuators (B_{ss}) is the most influential with regard to the second objective function as shown in Tables 6.6 and 6.7 and in Figs. 6.5 and 6.6. The higher magnitudes of W_s and B_{ss} result in better control performance characteristics. The effects of the actuator distance (Z), thickness (B_{ss}) and actuator length (d) appear to be negligible within the bounds considered when compared to the width and thickness in this case. The best design with respect to first objective function is noted from Tables 6.3 to be at : B_{ss} = 0.0005 m, W_s = 0.1 m, Z = 0.05 m and d = 0.23 m and the worst design is observed at : B_{ss} = 0.0005 m, W_s = 0.01 m, Z = 0.12 m and d = 0.26 m. The best design with respect to the second objective function is noted from Table 6.3 to be at : $B_{ss} = 0.0008$ m, $W_s = 0.01$ m, Z = 0.2 m and d = 0.23 m and the worst design is observed at : $B_{ss} = 0.0002$ m, $W_s = 0.1$ m, Z = 0.2 m and d = 0.26 m. Tip deflection plots of the closed-loop system are shown in Fig. 6.7 for the best and worst designs with respect to first objective function. The plots clearly indicate that the system at the best design performs much better.

6.2 Case 2

The master structure with a pair of piezoelectric actuators bonded to the whole top and bottom surfaces is shown in Case 2 of Fig. 6.1. The length and height of the structure, and the variation in the width of the structure and actuators are same as in the previous case, but the actuators which have only covered part of the structure now cover the whole length of the structure. The length of the actuators is same as that of the structure and the thickness of the actuators varies as before. The width and thickness of the actuator pairs (W_s and B_{ss}) are taken as the design variables affecting the control performance. The levels of design variables are listed in Table 6.8.

Taguchi methodology is applied to the system which is acted upon by step force and first type thermal disturbances. The L_9 OA, values of the objective functions and SNR are given in Table 6.9. The first two columns of the OA reflect the variation of the thickness and width of the actuators while the remaining two columns show the effect of the interaction between the thickness and the width of the actuators. The effects of different parameters (design variables and interactions together) are shown in Tables 6.9 through 6.11, and Figs. 6.8 and 6.9. It can be seen easily that with regard to the first objective function the width of the actuators (W_s) has a predominant effect on the control performance within the specified range. The thickness of the actuators (B_{ss}) has the similar dominant effect on the control performance of the system with respect to second objective function. The best design relative to first objective function is noted when $B_{ss} = 0.0008$ m and $W_s = 0.1$ m, and the worst design at the lower bounds, $B_{ss} = 0.0002$ m and $W_s = 0.01$ m. The best design relative to second objective function is noted when $B_{ss} = 0.0008$ m and $W_s = 0.01$ m, and the worst design at the lower bounds, $B_{ss} = 0.0002$ m and $W_s = 0.1$ m. The superiority of the best design over the worst design with respect to the first objective function is evident in the closed-loop system responses as shown in Fig. 6.10.

Lastly, Taguchi methodology is implemented on the system to which step force and second type thermal disturbances are imposed. The L_9 OA, design variables and levels are shown in Table 6.8 and the values of the objective function and SNR are given in Table 6.9. The effects of different parameters (design variables and interactions together) are shown in Tables 6.9, 6.12 and 6.13, and Figs. 6.11 and 6.12. It can be seen easily that the width of the actuators (W_s) has a predominant effect on the control performance within the specified range with respect to the first objective function. The thickness of the actuators (B_{ss}) has the similar dominant effect on the control performance with respect to the second objective function. The best design with respect to the first objective function is noted at the upper bounds, namely, $B_{ss} = 0.0008$ m and $W_s = 0.1$ m, and the worst design at the lower bounds, $B_{ss} = 0.0002$ m and $W_s = 0.01$ m. Same optimum design levels are obtained for the second objective function as in the previous case. The superiority of the best design over the worst design with respect to the first objective function is evident in the closed-loop system responses as shown in Fig. 6.13.

Table 6.1: Temperature Increase with Change in Dimensions of the Structure

NO.	Bss (m)	Ws (m)	θ_1 (K)	θ_2 (K)	θ_3 (K)	θ_4 (K)
1	0.0002	0.01	5.6	5.5	7.4	5.3
2	0.0002	0.05	14.5	14.5	15.8	13.9
3	0.0002	0.1	17.5	17.2	18	16.3
4	0.0005	0.01	6.5	6.3	10	6
5	0.0005	0.05	15	14.8	17	13.4
6	0.0005	0.1	17.8	17.6	20	16
7	0.0008	0.01	6.3	6.27	8.9	5
8	0.0008	0.05	16.4	16.2	17.8	15
9	0.0008	0.1	18.4	18.35	21.2	15.5

Table 6.2: Levels of Design Variables for Case 1 with Step Force and Thermal Disturbances

Bss (m)	Ws (m)	Z (m)	d (m)
0.0002	0.01	0.05	0.2
0.0005	0.05	0.12	0.23
0.0008	0.1	0.2	0.26

Table 6.3: L_9 Orthogonal Array, Objective Function and SNR Values for Case 1 with Step Force and Thermal Disturbances

Exp No.	Bss	Ws	Z	d	$(f_I)_1$	$(SNR_I)_1$	$(f_I)_2$	$(SNR_I)_2$	$(f_{II})_1$	$(SNR_{II})_1$	$(f_{II})_2$	$(SNR_{II})_2$
1	1	1	1	1	0.64	3.81	0.65	3.65	3.09E+11	-229.79	4.53E+11	-233.12
2	1	2	2	2	0.12	18.36	0.12	17.86	2.34E+12	-247.36	2.34E+12	-247.36
3	1	3	3	3	0.05	25.05	0.06	24.39	6.69E+12	-256.51	6.69E+12	-256.51
4	2	1	2	3	0.64	3.81	0.67	3.44	7.26E+11	-237.22	1.42E+12	-243.03
5	2	2	3	1	0.11	18.95	0.12	18.03	9.53E+11	-239.58	9.53E+11	-239.58
6	2	3	1	2	0.028	30.78	0.04	26.09	2.51E+12	-247.98	2.51E+12	-247.98
7	3	1	3	2	0.64	3.83	0.66	3.59	1.24E+10	-201.90	1.80E+10	-205.08
8	3	2	1	3	0.06	23.19	0.10	19.71	9.12E+10	-219.20	9.12E+10	-219.20
9	3	3	2	1	0.02	32.65	0.05	25.86	8.81E+10	-218.90	8.81E+10	-218.90

Table 6.4: ANOVA for Case 1 with Step Force and First Type Thermal Disturbances using First Objective Function

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S_x'	$\rho\%$
Bss_1	47.23	2	25.82	yes	2	25.82		12.91			
Bss_2	53.55										
Bss_3	59.68										
Ws_1	11.46	2	1013.40	no			506.70		67.15	998.31	94
Ws_2	60.51										
Ws_3	88.48										
Z_1	57.79	2	17.44	yes	2	17.44		8.72			
Z_2	54.83										
Z_3	47.83										
d_1	55.42	2	2.01	yes	2	2.01		1.00			
d_2	52.97										
d_3	52.06										
error											
e_{Total}			0		6	45.27		7.54			
Total			1058.67								

Table 6.5: ANOVA for Case 1 with Step Force and First Type Thermal Disturbances using Second Objective Function

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S'_x	$\rho\%$
Bss_1	-733.67	2	1782.31	no			891.15		31.82408	1726.29	72
Bss_2	-724.78										
Bss_3	-640.00										
Ws_1	-668.91	2	516.85	no			258.42		9.23	460.84	19
Ws_2	-706.15										
Ws_3	-723.39										
Z_1	-696.97	2	8.19	yes	2	8.19		4.09			
Z_2	-703.49										
Z_3	-697.99										
d_1	-688.27	2	103.83	yes	2	103.83		51.91			
d_2	-697.25										
d_3	-712.93										
error											
e_{Total}			0		4	112.02		28.00			
Total			2411.18								

Table 6.6: ANOVA for Case 1 with Step Force and Second Type Thermal Disturbances using First Objective Function

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S_x'	$\rho\%$
Bss_1	45.91	2	1.77	yes	2	1.77		0.88			
Bss_2	47.57										
Bss_3	49.17										
Ws_1	10.69	2	750.88	no			375.44		589.00	749.61	99
Ws_2	55.61										
Ws_3	76.34										
Z_1	49.46	2	2.04	yes	2	2.04		1.02			
Z_2	47.17										
Z_3	46.02										
d_1	47.55	2	2.6E-05	yes	2	2.6E-05		1.3E-05			
d_2	47.54										
d_3	47.55										
error											
e_{Total}			0		6	3.82		0.63			
Total			754.71								

Table 6.7: ANOVA for Case 1 with Step Force and Second Type Thermal Disturbances using Second Objective Function

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S_x'	$\rho\%$
Bss_1	-737.01	2	1831.46	no			915.733		25.41	1759.39	77
Bss_2	-730.59										
Bss_3	-643.19										
Ws_1	-681.24	2	299.39	no			149.69		4.15	227.32	10
Ws_2	-706.15										
Ws_3	-723.39										
Z_1	-700.31	2	16.41	yes	2	16.41		8.21			
Z_2	-709.30										
Z_3	-701.17										
d_1	-691.61	2	127.73	yes	2	127.73		63.86			
d_2	-700.43										
d_3	-718.74										
error											
e_{Total}			0		4	144.14		36.03			
Total			2275.01								

Table 6.8: Levels of Design Variables for Case 2 with Step Force and Thermal Disturbances

Bss (m)	Ws (m)
0.0002	0.01
0.0005	0.05
0.0008	0.1

Table 6.9: L_9 Orthogonal Array, Objective Function and SNR Values for Case 2 with Step Force and Thermal Disturbances

Exp No.	Bss	Ws	$B_{ss} \times W_s$	$B_{ss} \times W_s$	$(f_I)_1$	$(SNR_I)_1$	$(f_I)_2$	$(SNR_I)_2$	$(f_{II})_1$	$(SNR_{II})_1$	$(f_{II})_2$	$(SNR_{II})_2$
1	1	1	1	1	0.64	3.81	0.65	3.65	3.93E+12	-251.89	5.59E+12	-254.94
2	1	2	2	2	0.12	18.36	0.12	17.86	6.12E+12	-255.73	6.12E+12	-255.73
3	1	3	3	3	0.05	25.05	0.06	24.39	7.52E+12	-257.52	7.52E+12	-257.52
4	2	1	2	3	0.64	3.81	0.67	3.44	2.29E+11	-227.19	3.39E+11	-230.64
5	2	2	3	1	0.11	18.95	0.12	18.03	3.20E+11	-230.10	3.20E+11	-230.10
6	2	3	1	2	0.028	30.78	0.04	26.09	4.38E+11	-232.83	4.38E+11	-232.83
7	3	1	3	2	0.64	3.83	0.66	3.59	4.91E+10	-213.82	7.48E+10	-217.47
8	3	2	1	3	0.06	23.19	0.10	19.71	8.00E+10	-218.06	8.00E+10	-218.06
9	3	3	2	1	0.02	32.65	0.05	25.86	1.27E+11	-222.09	1.27E+11	-222.08

Table 6.10: ANOVA for Case 2 with Step Force and First Type Thermal Disturbances using First Objective Function

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S'_x	$\rho\%$
Bss_1	50.93	2	61.29	yes	2	61.29		30.64			
Bss_2	69.39										
Bss_3	64.65										
Ws_1	13.35	2	1321.45	no			660.72		24.71	1268.00	86
Ws_2	70.59										
Ws_3	101.03										
$(Bss \times Ws)_1$	74.70	2	85.16	yes	2	85.16		42.58			
$(Bss \times Ws)_2$	54.87										
$(Bss \times Ws)_3$	55.40										
$(Bss \times Ws)_1$	57.68	2	13.91	yes	2	13.91		6.95			
$(Bss \times Ws)_2$	66.65										
$(Bss \times Ws)_3$	60.65										
error											
e_{Total}			0		6	160.37		26.72			
Total			1481.83								

Table 6.11: ANOVA for Case 2 with Step Force and First Type Thermal Disturbances using Second Objective Function

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S'_x	$\rho\%$
Bss_1	-765.15	2	2144.30	no			1072.15		96.56	2122.10	96
Bss_2	-690.12										
Bss_3	-653.97										
Ws_1	-692.91	2	63.91	yes	2	63.91		31.95			
Ws_2	-703.90										
Ws_3	-712.44										
$(Bss \times Ws)_1$	-702.78	2	2.17	yes	2	2.17		1.08			
$(Bss \times Ws)_2$	-705.02										
$(Bss \times Ws)_3$	-701.44										
$(Bss \times Ws)_1$	-704.08	2	0.53	yes	2	0.53		0.26			
$(Bss \times Ws)_2$	-702.38										
$(Bss \times Ws)_3$	-702.77										
error											
e_{Total}			0		6	66.61		11.10			
Total			2210.92								

Table 6.12: ANOVA for Case 2 with Step Force and Second Type Thermal Disturbances using First Objective Function

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S'_x	$\rho\%$
Bss_1	48.41	2	57.48	yes	2	57.48		28.74			
Bss_2	55.54										
Bss_3	66.82										
Ws_1	14.83	2	1048.78	no			524.39		38.78	1021.75	90
Ws_2	62.33										
Ws_3	93.61										
$(Bss \times Ws)_1$	57.26	2	12.16	yes	2	12.16		6.08			
$(Bss \times Ws)_2$	61.01										
$(Bss \times Ws)_3$	52.49										
$(Bss \times Ws)_1$	61.70	2	11.46	yes	2	11.46		5.73			
$(Bss \times Ws)_2$	54.79										
$(Bss \times Ws)_3$	54.28										
error											
e_{Total}			0		6	81.11		13.52			
Total			1129.90								

Table 6.13: ANOVA for Case 2 with Step Force and Second Type Thermal Disturbances using Second Objective Function

Factor level	SNR_{Total}	df	S_x	pool	df_e	S_e	V_x	V_e	F	S_x'	$\rho\%$
Bss_1	-768.21	2	21	no			1060.68		313.60	2114.61	99
Bss_2	-693.54										
Bss_3	-657.62										
Ws_1	-703.03	2	18.00	yes	2	18.00		9.00			
Ws_2	-703.90										
Ws_3	-712.43										
$(Bss \times Ws)_1$	-705.83	2	2.05	yes	2	2.05		1.025			
$(Bss \times Ws)_2$	-708.44										
$(Bss \times Ws)_3$	-705.09										
$(Bss \times Ws)_1$	-707.13	2	0.23	yes	2	0.23		0.11			
$(Bss \times Ws)_2$	-706.04										
$(Bss \times Ws)_3$	-706.19										
error											
e_{Total}			0		6	20.29		3.38			
Total			2141.67								

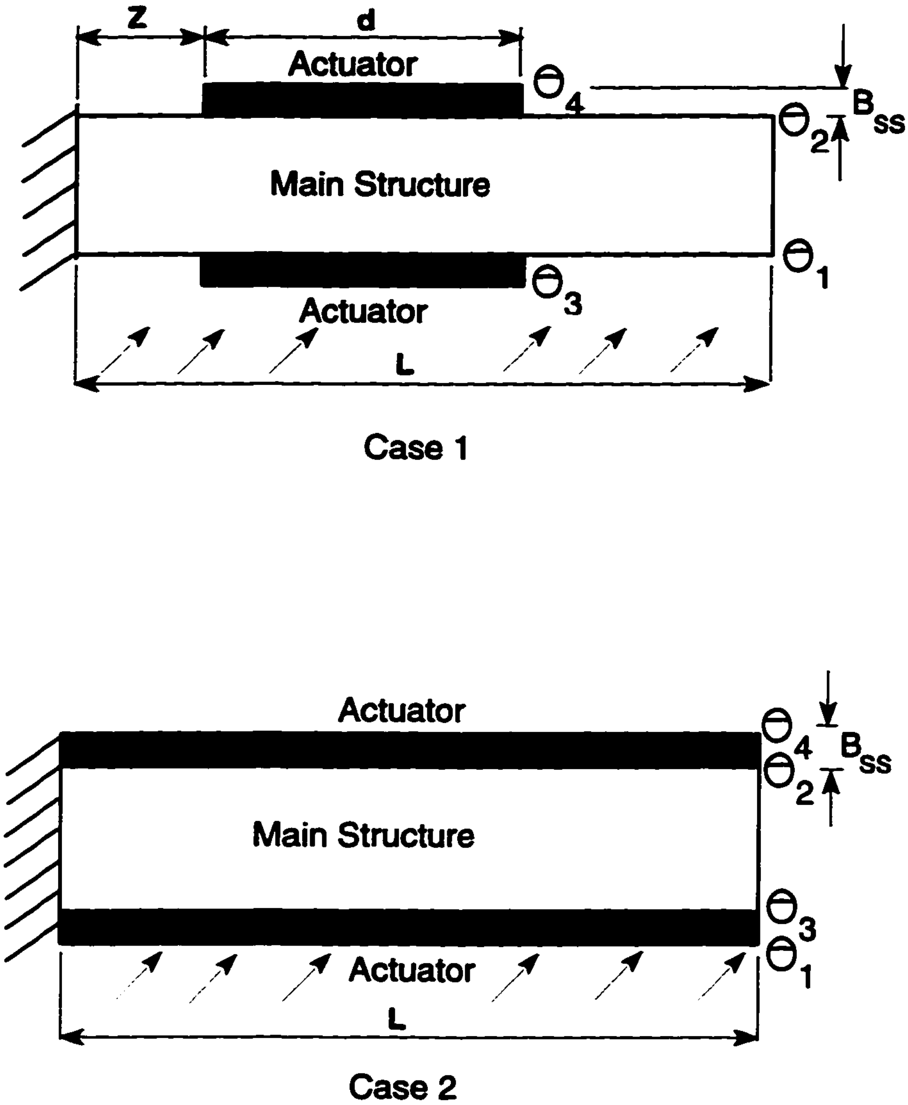

Figure 6.1: Thermopiezoelectrical Control Systems for Different Cases

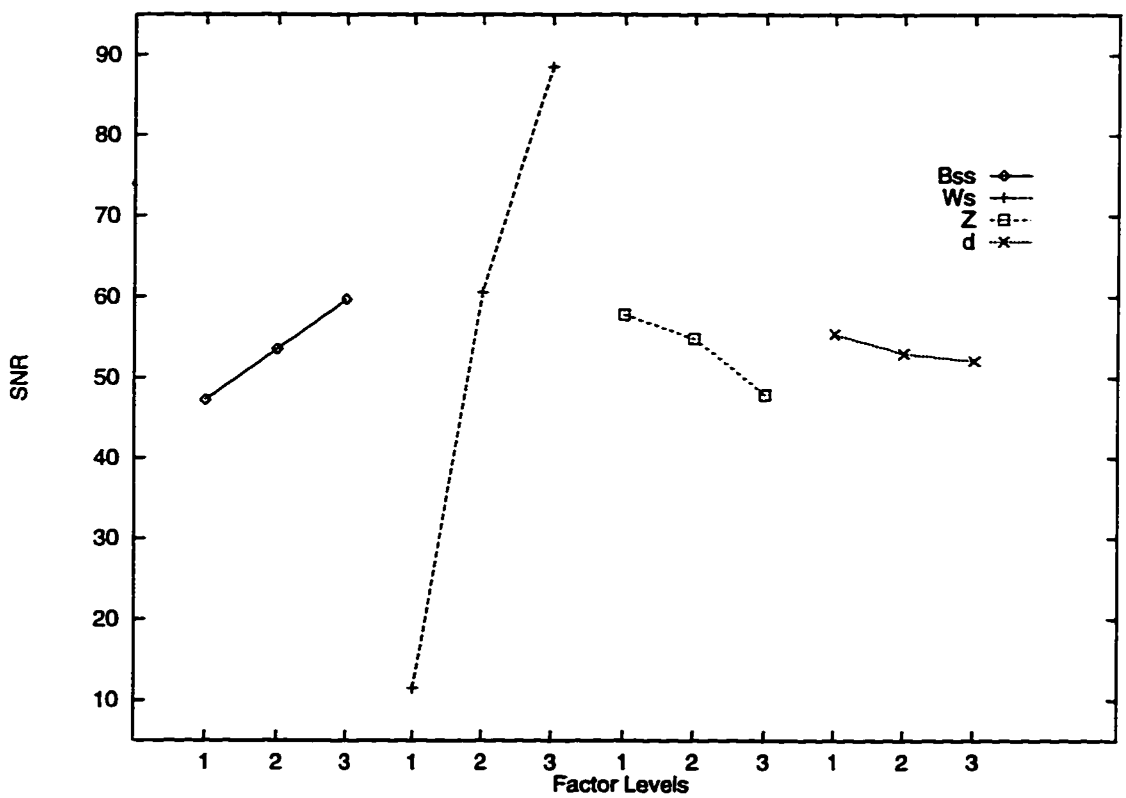

Figure 6.2: Effect of Each Design Variable on Objective Function for Case 1 with Step Force and First Type Thermal Disturbances using First Objective Function

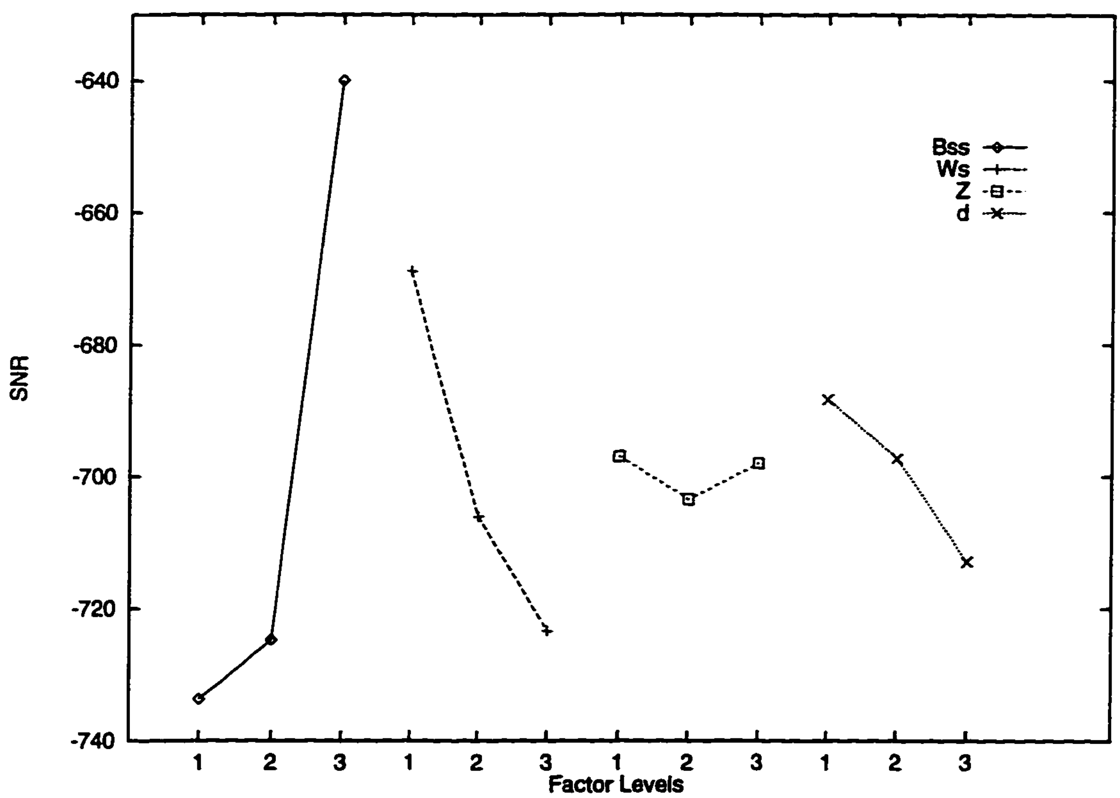

Figure 6.3: Effect of Each Design Variable on Objective Function for Case 1 with Step Force and First Type Thermal Disturbances using Second Objective Function

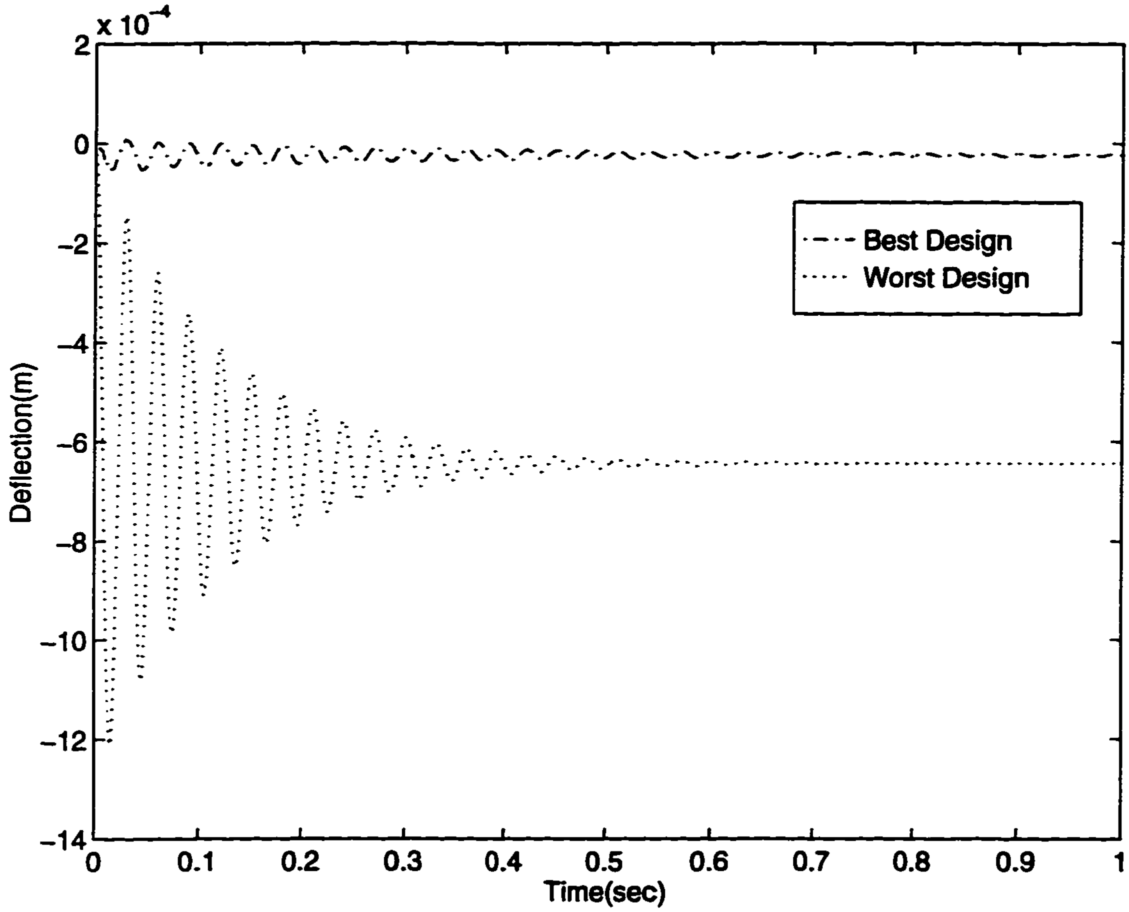

Figure 6.4: Tip Deflection of Closed-Loop System for Case 1 with Step Force and First Type Thermal Disturbances with First Objective Function

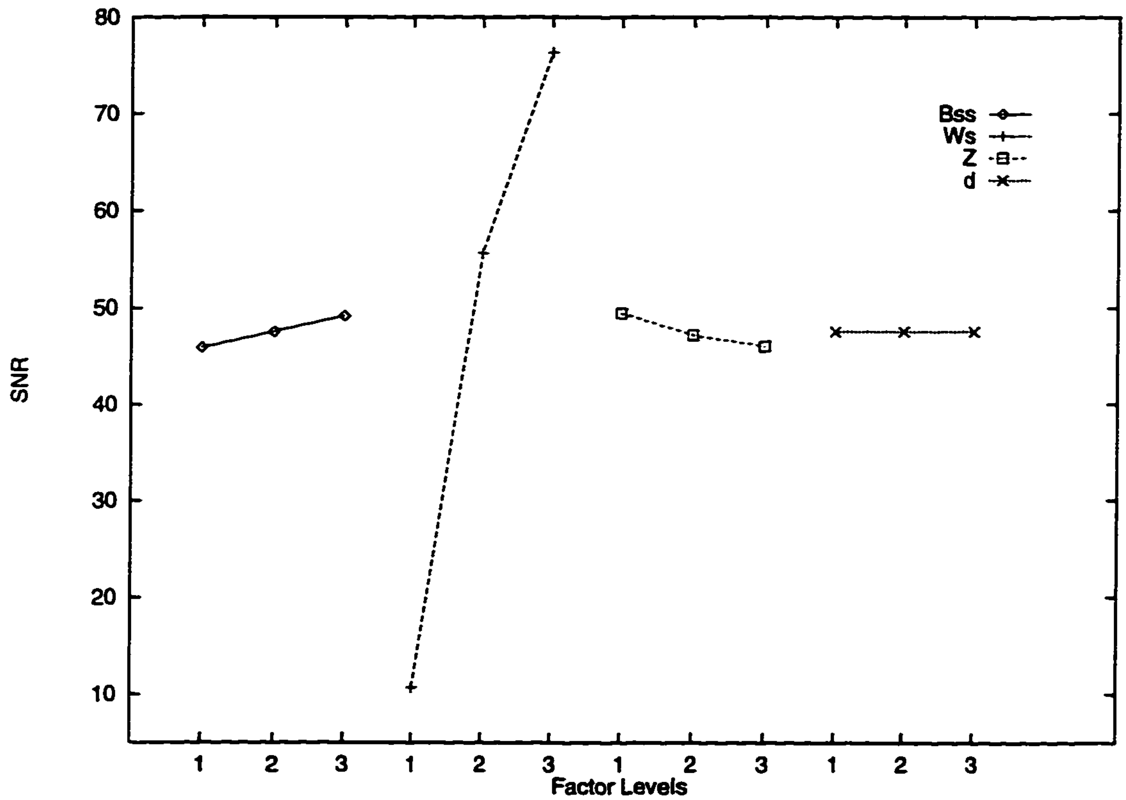

Figure 6.5: Effect of Each Design Variable on Objective Function for Case 1 with Step Force and Second Type Thermal Disturbances using First Objective Function

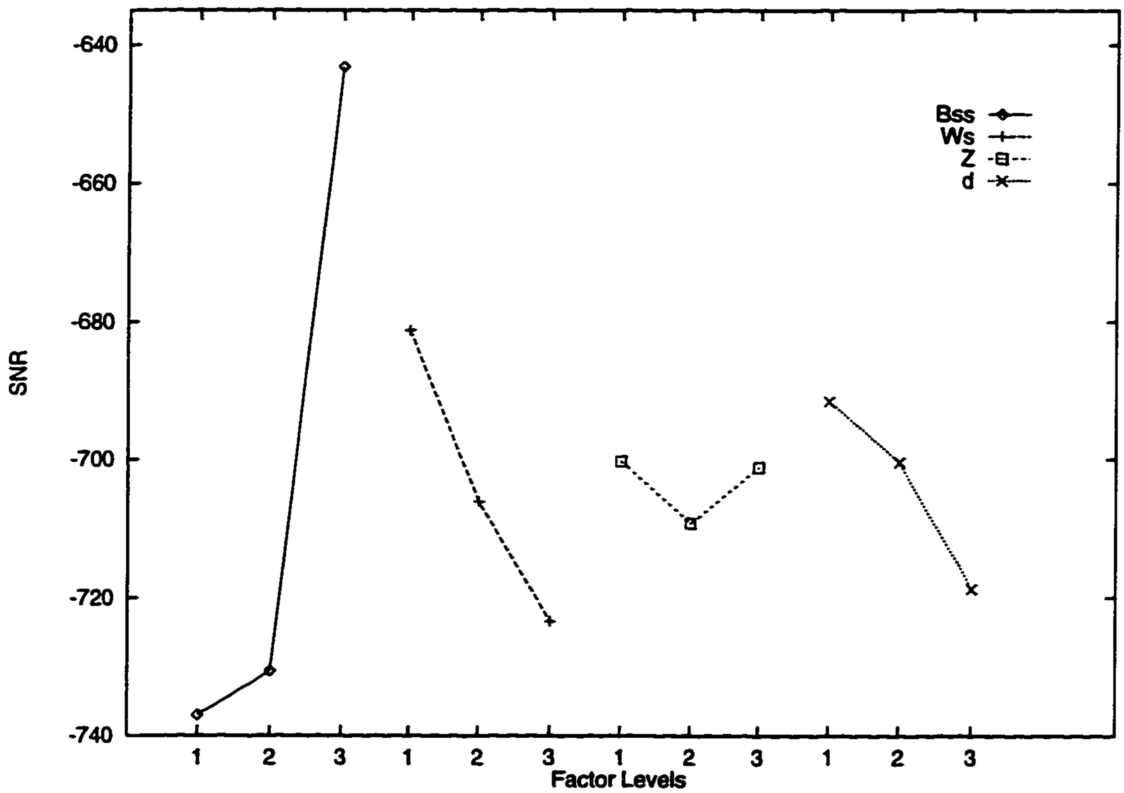

Figure 6.6: Effect of Each Design Variable on Objective Function for Case 1 with Step Force and Second Type Thermal Disturbances using Second Objective Function

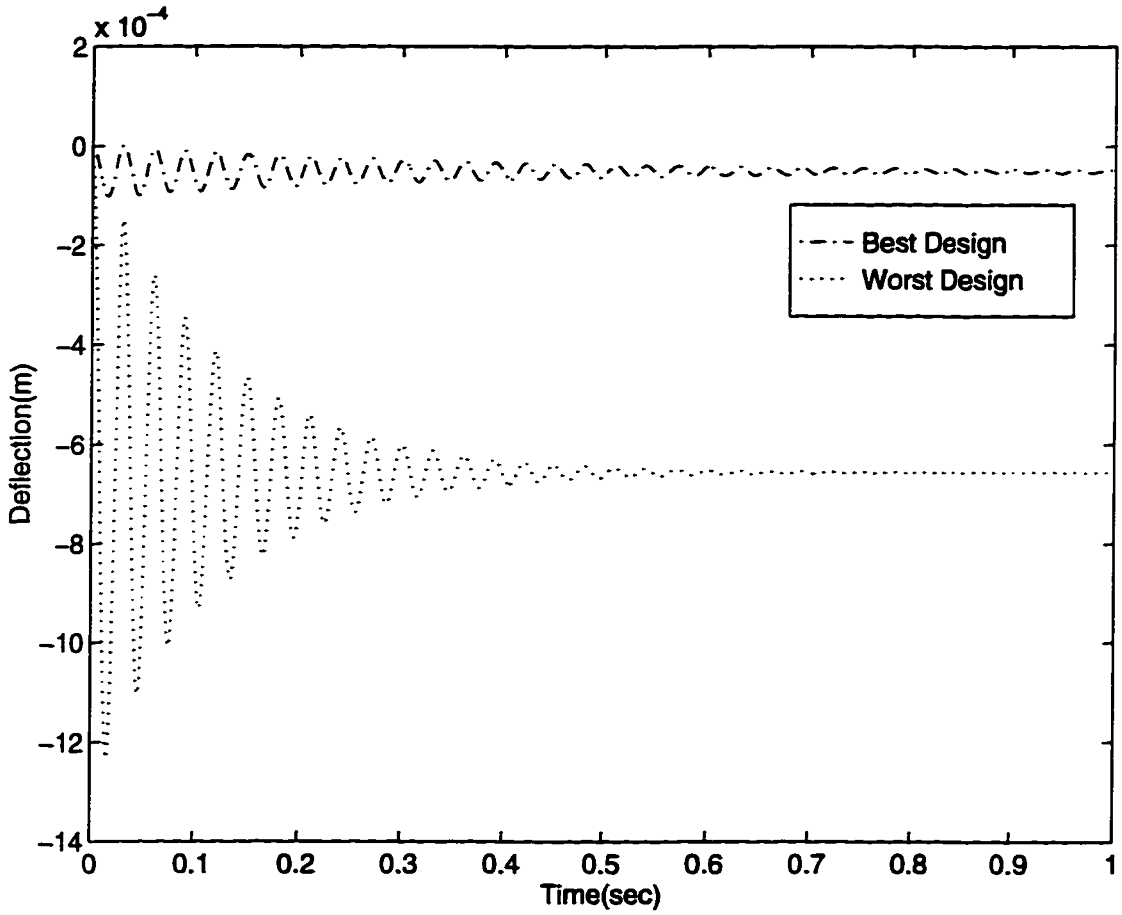

Figure 6.7: Tip Deflection of Closed-Loop System for Case 1 with Step Force and Second Type Thermal Disturbances with First Objective Function

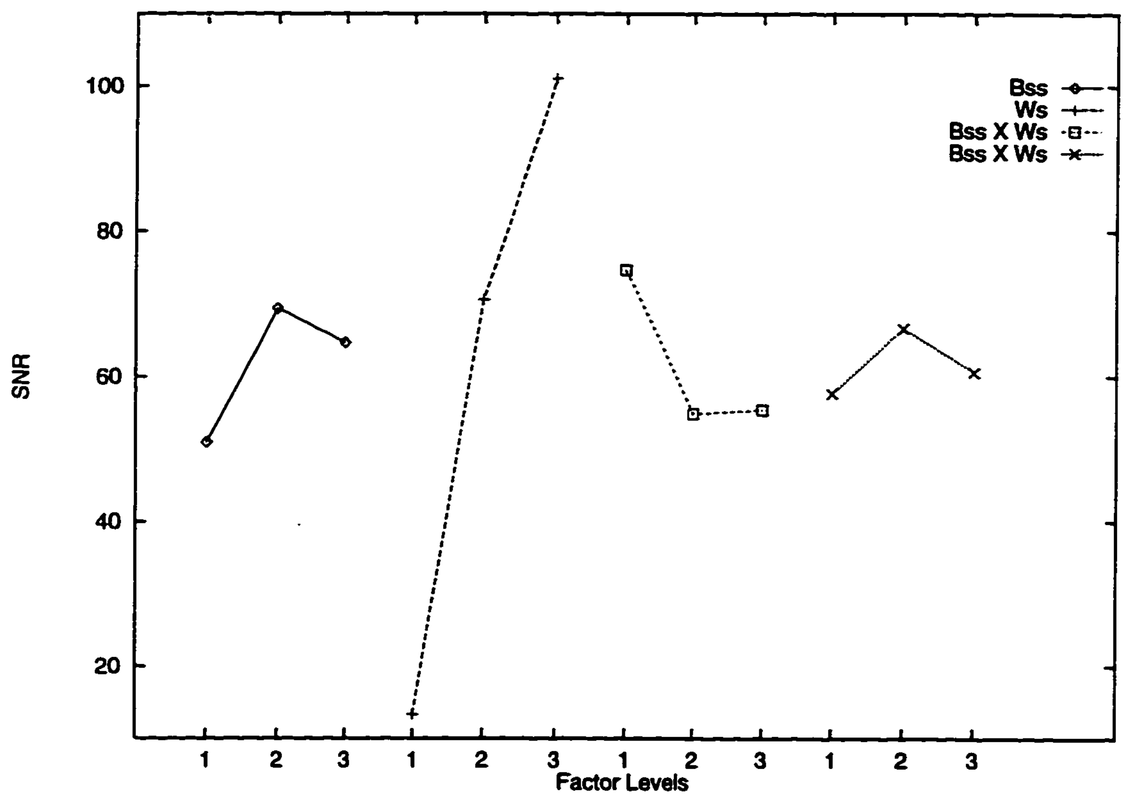

Figure 6.8: Effect of Each Factor on Objective Function for Case 2 with Step Force and First Type Thermal Disturbances using First Objective Function

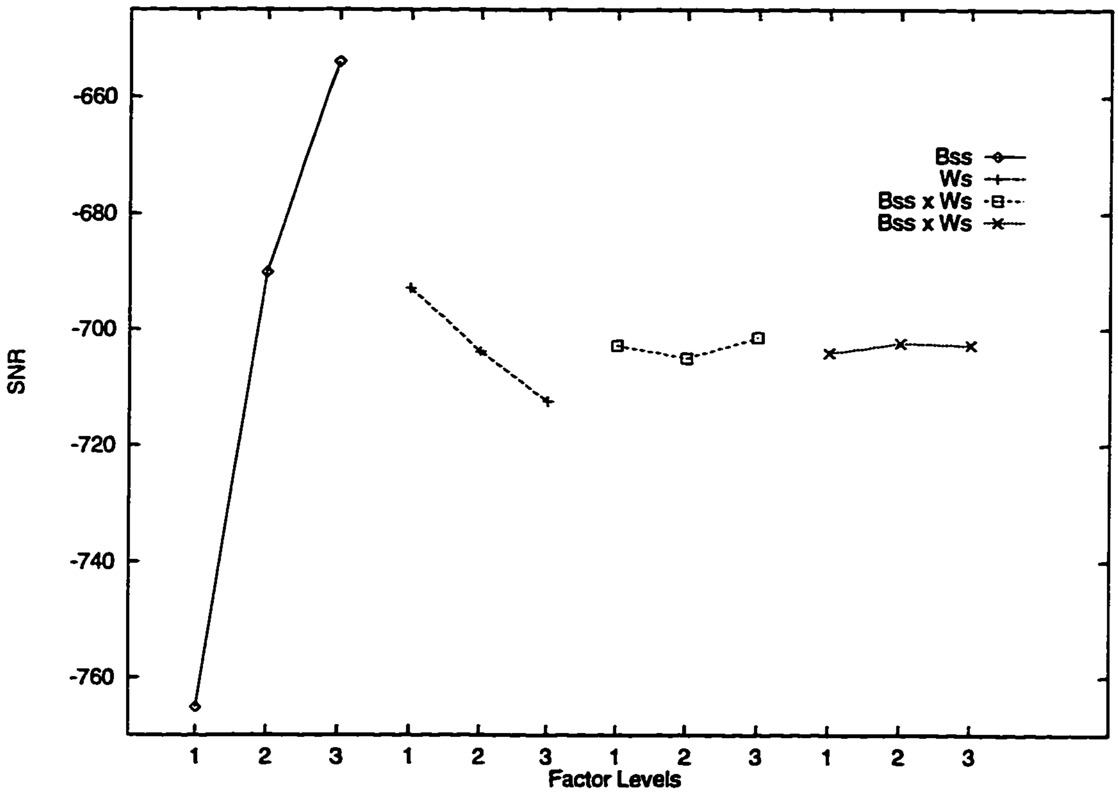

Figure 6.9: Effect of Each Factor on Objective Function for Case 2 with Step Force and First Type Thermal Disturbances using Second Objective Function

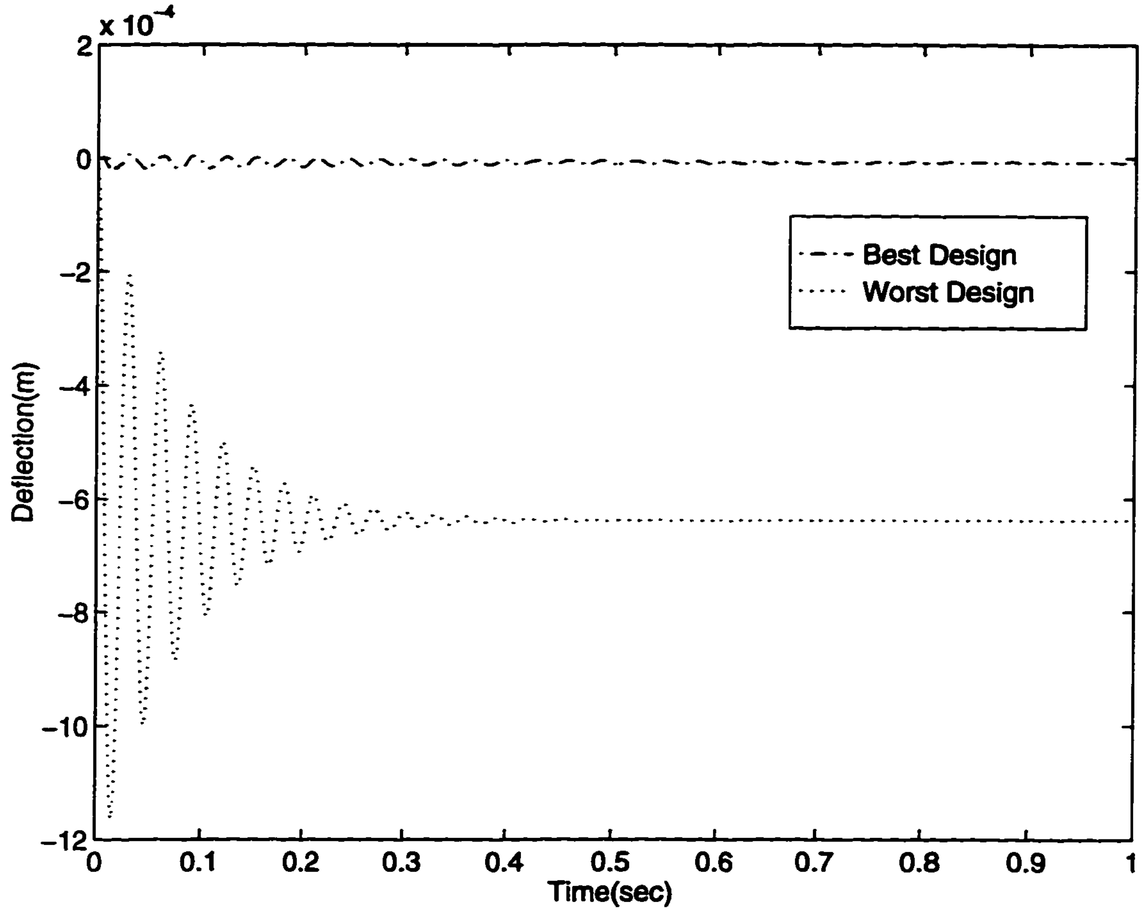

Figure 6.10: Tip Deflection of Closed-Loop System for Case 2 with Step Force and First Type Thermal Disturbances with First Objective Function

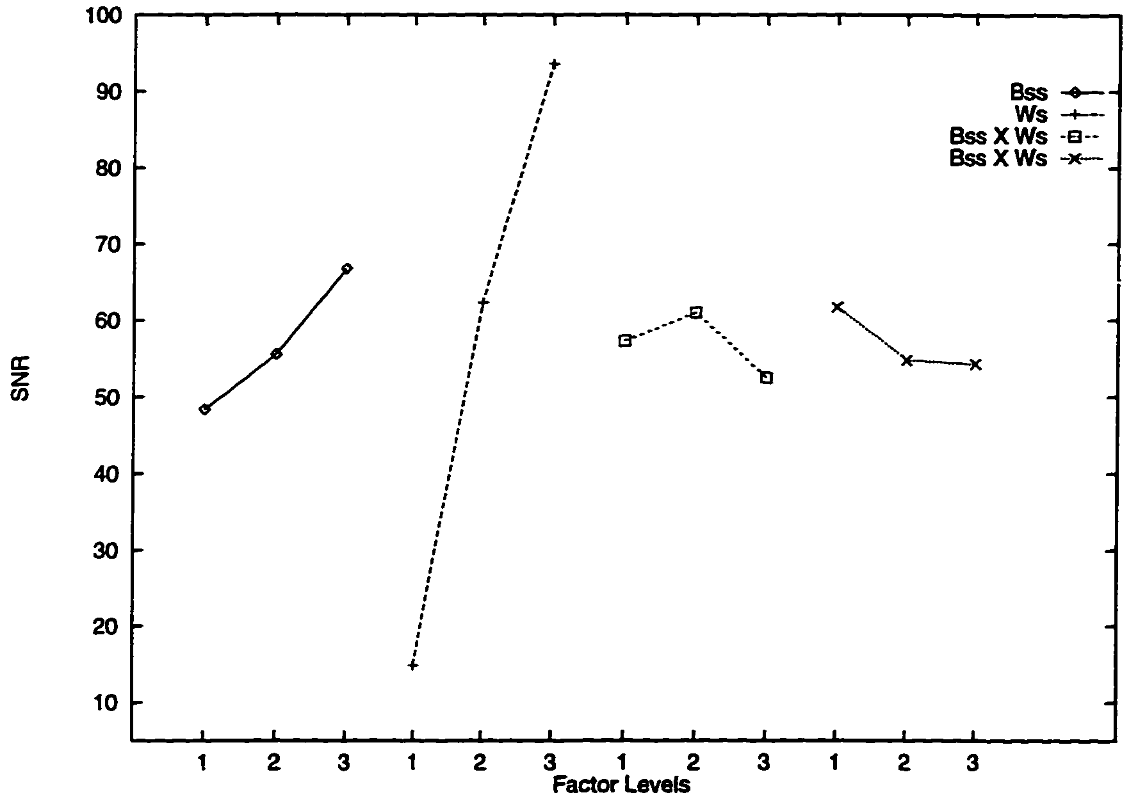

Figure 6.11: Effect of Each Factor on Objective Function for Case 2 with Step Force and Second Type Thermal Disturbances using First Objective Function

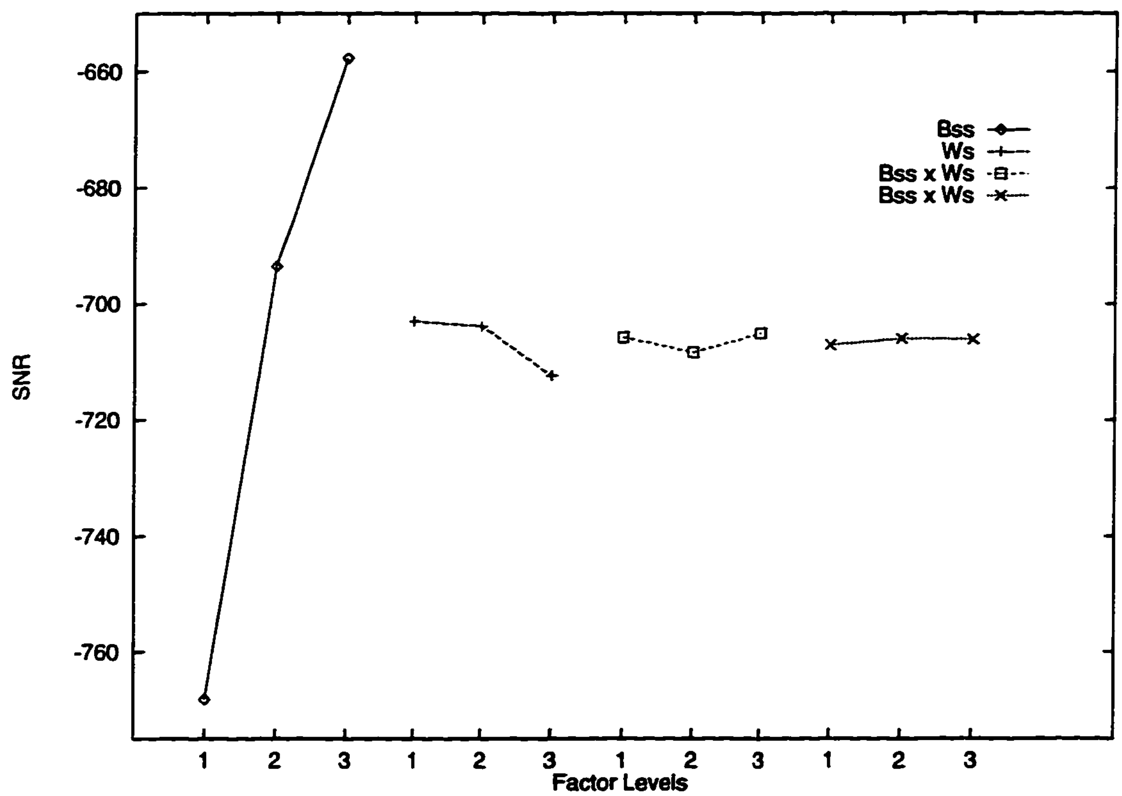

Figure 6.12: Effect of Each Factor on Objective Function for Case 2 with Step Force and Second Type Thermal Disturbances using Second Objective Function

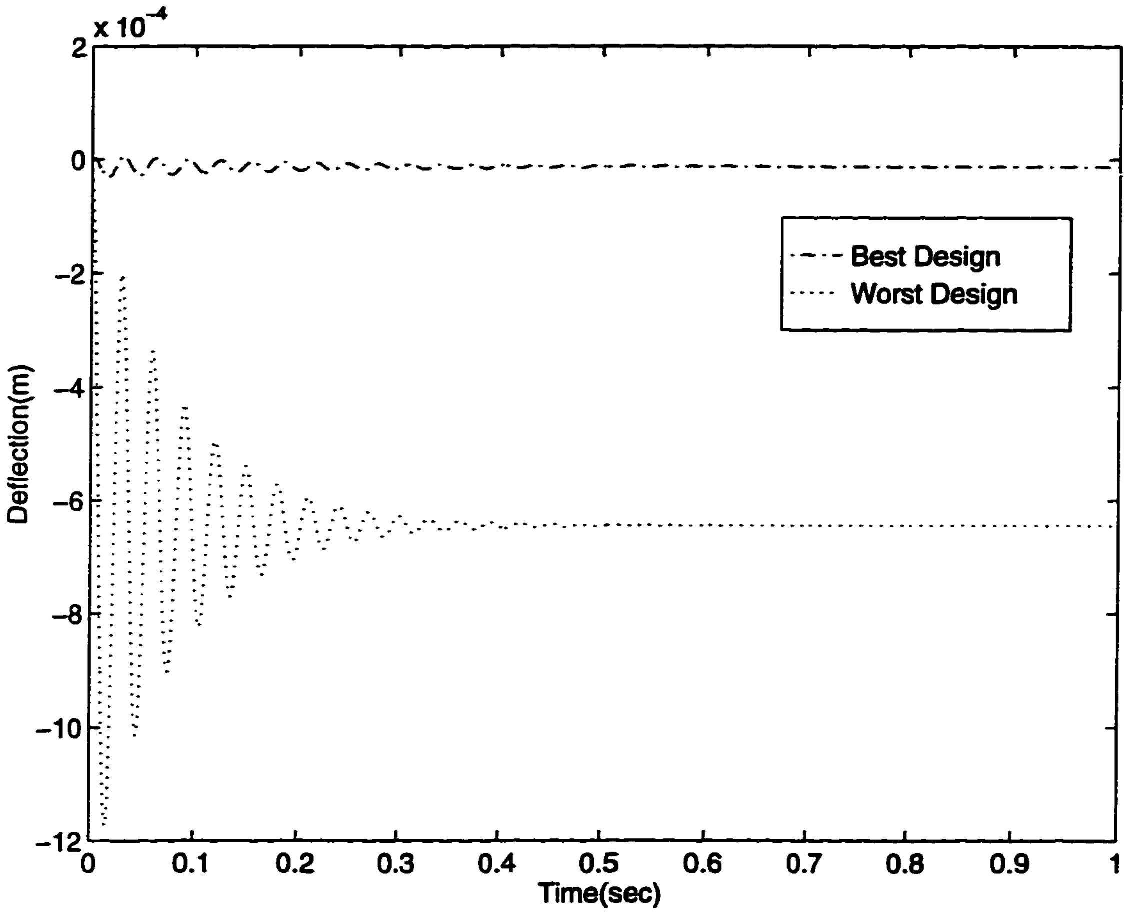

Figure 6.13: Tip Deflection of Closed-Loop System for Case 2 with Step Force and Second Type Thermal Disturbances with First Objective Function

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

Finite Element modelling of piezoelectric materials is presented using the quasi-static piezoelectric equations in Hamilton's principle. Taguchi robust design of piezoelectric actuators is carried out on three different configurations of piezoelectric control systems and two configurations of thermopiezoelectric control systems in order to determine the parameters affecting the control performance and their levels of influence. A novel optimization scheme is designed to compare the results with those of Taguchi methodology.

Based on Taguchi robust design methodology, it is concluded that the thickness and width of the piezoelectric actuator pairs have varying effects on the control performance of cantilever beam-like structures subjected to vibratory and thermal disturbances.

- The thickness appears to have greater impact when compared to the width when subjected to impulse as the input, on the other hand when subjected to step as input the width has the most predominant effect.

- As evident from the results of Taguchi methodology and optimization scheme, the actuator locations also influence the control performance.

- It can be concluded from the simulations that for the first objective function highest dimensions of thickness and width results in better control performance while for the second objective function the system performs best for highest dimension of thickness and lowest dimension of width.

- As for location and length of the actuators the system performs best when the actuators are close to the fixed end and the length of the actuators is small for the first objective function whereas the control performance increases when the actuators are away from the fixed end and length of the actuators is large for second objective function.

Within the specified ranges taken in this study, it can be concluded in general that depending on the objective funtion one has to select suitable dimensions to acheive better control performance. As far as minimization of deflections is concerned highest dimensions of thickness and width are preferred whereas if the objective is to minimize the effective damping time then highest dimesions of thickness and lowest dimension of width helps in attenuating structural and thermal disturbances.

7.2 Recommedations

- It will be helpful to study the magnetic effects in piezoelectric media.
- Other objective functions can be defined in Taguchi methodology for the extension of this research.

APPENDIX

Matlab Program for Case 1

```
function [f,t,y]=piezt(obj,form,b2,w1,z,d,th)
v=250;
l=0.5;ndof=28;ndofpa=28;nnodfa=6;nelem=8;nelema=4;
ass(1)=z/2;ass(2)=d/4;ass(3)=d/4;ass(4)=(l-d-z)/2;
ass(5)=d/4;ass(6)=d/4;ass(7)=d/4;ass(8)=d/4;
bss(1)=0.005;bss(2)=0.005;bss(3)=0.005;bss(4)=0.005;
bss(5)=b2/2;bss(6)=b2/2;bss(7)=b2/2;bss(8)=b2/2;
ws=w1;
rhosn=[2750,2750,2750,2750,1800,1800,1800,1800];
if th == 0
econ=3.8e9;
else
econ=2.6e9;
end
e=[7.3e10,7.3e10,7.3e10,7.3e10,econ,econ,econ,econ];
anu=[0.33,0.33,0.33,0.33,0.29,0.29,0.29,0.29];
if th == 0
el31con=0.046;
else
el31con=0.047;
end
el31n=[-el31con,-el31con,el31con,el31con];
al1=0;
if th == 0
alp3con=1.5e-4;
else alp3con=1.75e-4;
end
alp3=[2.4e-5,2.4e-5,2.4e-5,2.4e-5,alp3con,alp3con];
alp3=[alp3,alp3con,alp3con];
ap=d/4;bp=b2/2;wp=w1;rhop=1800;el15=0;el33=0;al13=0;
eps11=0;nelcaf=[1,2,13,13;2,3,13,13;13,13,5,4;13,13,6,5];
nnodta=16;cp=3.8e-6;theta0=300;
if th == 0
p3con=4e-5;
eps33con=1.026e-10;
else
p3con=4.5e-5;
```

```
eps33con=1.026e-10;
end
p3n=[p3con,p3con,p3con,p3con];
eps33n=[eps33con,eps33con,eps33con,eps33con];
ak11=0.52;ak33=0.12;
nelcat=[1,2,7,6;2,3,8,7;3,4,9,8;4,5,10,9;
11,12,3,2;12,13,4,3;7,8,15,14;8,9,16,15];
iden=[50,50,1,2,9,10,50,50;1,2,3,4,11,12,9,10;3,4,5,6,13,14,11,12;
5,6,7,8,15,16,13,14;17,18,19,20,3,4,1,2;19,20,21,22,5,6,3,4;
9,10,11,12,25,26,23,24;11,12,13,14,27,28,25,26];
idenpa=[17,18,19,20,3,4,1,2;19,20,21,22,5,6,3,4;
9,10,11,12,25,26,23,24;11,12,13,14,27,28,25,26];
af=[-v,-v,-v,v,v,v]';
at=[th,th,th,th,th,th,th,th,th,th,th,th,th,th,th,th]';
i=1;
while i≤ nelem
epss(i)=e(i)/(1-anu(i)×anu(i));pss1(i)=0.5×(1-anu(i));
epst(i)=e(i)/(1+anu(i))/(1-2×anu(i));
pst1(i)=1-anu(i);pst2(i)=0.5×(1-2×anu(i));e11n(i)=epss(i);
e22n(i)=epss(i);al3n(i)=alp3(i)×e22n(i);
e12n(i)=epss(i)×anu(i);e33n(i)=epss(i)×pss1(i);
i=i+1;
end
i=1;
while i ≤ ndof
j=1;
while j ≤ ndof
akuu(i,j)=0;
amuu(i,j)=0;
j=j+1;
end
i=i+1;
end
i=1;
while i ≤ ndofpa
j=1;
while j ≤ nnodfa
akufa(i,j)=0;
j=j+1;
end
i=i+1;
```

```
end
i=1;
while i ≤ ndofpa
j=1;
while j ≤ nnodta
akuta(i,j)=0;
j=j+1;
end
i=i+1;
end
i=1;
while i ≤ nnodta
j=1;
while j ≤ ndofpa
actua(i,j)=0;
j=j+1;
end
i=i+1;
end
i=1;
while i ≤ nnodfa
j=1;
while j ≤ nnodfa
akffa(i,j)=0;
j=j+1;
end
j=1;
while j ≤ nnodta
akfta(i,j)=0;
j=j+1;
end
i=i+1;
end
i=1;
while i ≤ nnodta
j=1;
while j ≤ nnodfa
actfa(i,j)=0;
j=j+1;
end
i=i+1;
```

```
end
i=1;
while i ≤ nnodta
j=1;
while j ≤ nnodta
aktta(i,j)=0;
j=j+1;
end
i=i+1;
end
n=1;
while n ≤ nelem
as=ass(n);
bs=bss(n);
rhos=rhosn(n);
e11=e11n(n);
e22=e22n(n);
e12=e12n(n);
e33=e33n(n);
```

$$[akuuel, amuuel] = elems(as, bs, ws, rhos, e11, e22, e12, e33);$$

```
i=1;
while i ≤ 8
i1=iden(n,i);
while i1 ≤ ndof
j=1;
while j ≤ 8
j1=iden(n,j);
while j1 ≤ ndof
akuu(i1,j1)=akuu(i1,j1)+akuuel(i,j);
amuu(i1,j1)=amuu(i1,j1)+amuuel(i,j);
j1=ndof+1;
end
j=j+1;
end
i1=ndof+1;
end
i=i+1;
end
n=n+1;
end
```

```
n=1;
while n ≤ nelema
el31=el31n(n);
p3=p3n(n);
eps33=eps33n(n);
```

$$[akufel, akffel, akftel, actfel] = elempf(ap, bp, wp, rhop, el15, el31, el33, eps11, eps33, p3, theta0);$$

```
i=1;
while i ≤ 8
i1=idenpa(n,i);
while i1 ≤ ndofpa
j=1;
while j ≤ 4
jaf=nelcaf(n,j);
while jaf ≤ nnodfa
akufa(i1,jaf)=akufa(i1,jaf)+akufel(i,j);
jaf=nnodfa+1;
end
j=j+1;
end
i1=ndofpa+1;
end
i=i+1;
end
i=1;
while i ≤ 4
iat=nelcat(n,i);
while i1 ≤ nnodta
j=1;
while j ≤ 4
jaf=nelcaf(n,j);
while jaf ≤ nnodfa
actfa(iat,jaf)=actfa(iat,jaf)+actfel(i,j);
jaf=nnodfa+1;
end
j=j+1;
end
iat=nnodta+1;
end
```

```
i=i+1;
end
i=1;
while i ≤ 4
iaf=nelcaf(n,i);
while iaf ≤ nnodfa
j=1;
while j ≤ 4
jaf=nelcaf(n,j);
while jaf ≤ nnodfa
akffa(iaf,jaf)=akffa(iaf,jaf)+akffel(i,j);
jaf=nnodfa+1;
end
j=j+1;
end
j=1;
while j ≤ 4
jat=nelcat(n,j);
while jat ≤ nnodta
akfta(iaf,jat)=akfta(iaf,jat)+akftel(i,j);
jat=nnodta+1;
end
j=j+1;
end
iaf=nnodfa+1;
end
i=i+1;
end
n=n+1;
end
n=1;
while n ≤ nelem
as=ass(n);
bs=bss(n);
al3=al3n(n);
```

$$[akutel, actuel, akttel] = elemt(as, bs, ws, al1, al3, al13, theta0, ak11, ak33);$$

```
i=1;
while i ≤ 8
i1=iden(n,i);
while i1 ≤ ndof
```

```
j=1;
while j ≤ 4
jat=nelcat(n,j);
while jat ≤ nnodta
akuta(i1,jat)=akuta(i1,jat)+akutel(i,j);
jat=nnodta+1;
end
j=j+1;
end
i1=ndof+1;
end
i=i+1;
end
i=1;
while i ≤ 4
iat=nelcat(n,i);
while i1 ≤ nnodta
j=1;
while j ≤ 4
jat=nelcat(n,j);
while jat ≤ nnodta
aktta(iat,jat)=aktta(iat,jat)+akttel(i,j);
jat=nnodta+1;
end
j=j+1;
end
j=1;
while j ≤ 8
j1=iden(n,j);
while j1 ≤ ndof
actua(iat,j1)=actua(iat,j1)+actuel(i,j);
j1=ndof+1;
end
j=j+1;
end
iat=nnodta+1;
end
i=i+1;
end
n=n+1;
end
```

```
u=inv(akuu)×(-akufa×af+akuta×at);
force=akuta×at;
phi=inv(akffa+akufa'×inv(akuu)×akufa)×(akfta+akufa'×inv(akuu)×akuta)×at;
ak1=akuu+akufa×inv(akffa)×akufa';
d=cp×akufa×inv(akffa);
cc1=10;cc2=10^-11;
acuu=cc1×amuu+cc2×akuu;
```

$$a = [zeros(ndof), eye(ndof); -inv(amuu) \times ak1, zeros(ndof)];$$

```
b=[0×d;inv(amuu)×d];
q=100×eye(2×ndofpa);r=eye(nnodfa);
```

$$[k, s] = lqr(a, b, q, r);$$

```
acl=a-b×k;
df=[zeros(1,15),-1,zeros(1,12)]';bf=[0×df;inv(amuu)×df];
dtte=akuta×at;dtpy=-akufa×inv(akffa)×akfta×at;dt=dtte+dtpy;
bt=[0×dt;inv(amuu)×dt];
b=bf+bt;
c=[zeros(1,15),1,zeros(1,40)];d=0;
t=0:0.001:1;
a=[zeros(ndof),eye(ndof);-inv(amuu)×ak1,-inv(amuu)×acuu];
if form ≤ 1
y=step(acl,b,c,d,1,t);
else
y=0.01×impulse(acl,b,c,d,1,t);
end
ysste=inv(ak1)×dtte;ysspy=inv(ak1)×dtpy;
yss=inv(ak1)×df;
if obj≤1
i=1;f=0;
while i ≤ 1001
f=f+abs(y(i));
i=i+1;
end
else
f=trace(s)/trace(q);
end
end
```

Function Elems for Case 1

```
function [akels,amuuel]=elems(a,b,t,rho,e11,e22,e12,e33)
s1=t×b×e11/6./a;
s2=t×a×e22/6./b;
s3=t×e12/4.;
s4=t×a×e33/6./b;
s5=t×b×e33/6./a;
s6=t×e33/4.;
akel(1,1)=2.×(s1+s4);
akel(2,1)=s3+s6;
akel(2,2)=2.×(s2+s5);
akel(3,1)=s4-2.×s1;
akel(3,2)=s6-s3;
akel(3,3)=2.×(s1+s4);
akel(4,1)=s3-s6;
akel(4,2)=s2-2.×s5;
akel(4,3)=-s3-s6;
akel(4,4)=2.×(s2+s5);
akel(5,1)=-s1-s4;
akel(5,2)=-s3-s6;
akel(5,3)=s1-2.×s4;
akel(5,4)=s6-s3;
akel(5,5)=2.×(s1+s4);
akel(6,1)=-s3-s6;
akel(6,2)=-s2-s5;
akel(6,3)=s3-s6;
akel(6,4)=s5-2.×s2;
akel(6,5)=s3+s6;
akel(6,6)=2.×(s2+s5);
akel(7,1)=s1-2.×s4;
akel(7,2)=s3-s6;
akel(7,3)=-s1-s4;
akel(7,4)=s3+s6;
akel(7,5)=s4-2.×s1;
akel(7,6)=s6-s3;
akel(7,7)=2.×(s1+s4);
akel(8,1)=s6-s3;
akel(8,2)=s5-2.×s2;
akel(8,3)=s3+s6;
akel(8,4)=-s2-s5;
akel(8,5)=s3-s6;
```

```
akel(8,6)=s2-2.×s5;
akel(8,7)=-s3-s6;
akel(8,8)=2.×(s2+s5);
i=1;
while i ≤ 8
j=1;
while j ≤ 8
if i ¿ j
akel(j,i)=akel(i,j);
end
j=j+1;
end
i=i+1;
end
i=1;
while i ≤ 8
j=1;
while j ≤ 8
akeln(i,j)=0;
j=j+1;
end
i=i+1;
end
akeln(1,1)=1;
akeln(3,1)=-1;
akeln(5,1)=1;
akeln(7,1)=-1;
akeln(1,3)=-1;
akeln(3,3)=1;
akeln(5,3)=-1;
akeln(7,3)=1;
akeln(1,5)=1;
akeln(3,5)=-1;
akeln(5,5)=1;
akeln(7,5)=-1;
akeln(1,7)=-1;
akeln(3,7)=1;
akeln(5,7)=-1;
akeln(7,7)=1;
say=t×(e33×a×a+e12×e12×b×b/e22)/12./a/b;
i=1;
```

```
while i ≤ 8
j=1;
while j ≤ 8
akeln(i,j)=say×akeln(i,j);
j=j+1;
end
i=i+1;
end
i=1;
while i ≤ 8
j=1;
while j ≤ 8
akels(i,j)=akel(i,j)-akeln(i,j);
j=j+1;
end
i=i+1;
end
i=1;
while i ≤ 8
j=1;
while j ≤ 8
amuuel(i,j)=0;
j=j+1;
end
i=i+1;
end
i=1;
while i ≤ 8
amuuel(i,i)=4;
i=i+1;
end
i=1;
while i ≤ 6
amuuel(i+2,i)=2;
amuuel(i,i+2)=2;
i=i+1;
end
i=1;
while i ≤ 4
amuuel(i+4,i)=1;
amuuel(i,i+4)=1;
```

```
i=i+1;
end
i=1;
while i ≤ 2
amuuel(i+6,i)=2;
amuuel(i,i+6)=2;
i=i+1;
end
amass=rho×a×b×t/9;
i=1;
while i ≤ 8
j=1;
while j ≤ 8
amuuel(i,j)=amass×amuuel(i,j);
j=j+1;
end
i=i+1;
end
end
```

Function Elempf for Case 1

```
function [akufel,akffel,akftel,actfel]=elempf(a,b,t,rhop,el15,el31,el33,eps11,eps33,p3,theta0)
c1=b/a/3;
c2=a/b/3;
c3=c1/2;
c4=c2/2;
akftel(1,1)=-2;
akftel(1,2)=-1;
akftel(1,3)=-1;
akftel(1,4)=-2;
akftel(2,1)=-1;
akftel(2,2)=-2;
akftel(2,3)=-2;
akftel(2,4)=-1;
i=1;
while i ≤ 4
akftel(3,i)=-akftel(2,i);
akftel(4,i)=-akftel(1,i);
i=i+1;
end
cft=t×a×p3/6;
i=1;
while i ≤ 4
j=1;
while j ≤ 4
akftel(i,j)=cft×akftel(i,j);
j=j+1;
i=i+1;
end
actfel=theta0×akftel';
c1=b/a/3;
c2=a/b/3;
c3=b/a/6;
c4=a/b/6;
akufel(1,1)=0.25×(el15+el31);
akufel(1,2)=0.25×(-el15+el31);
akufel(1,3)=-0.25×(el15+el31);
akufel(1,4)=0.25×(el15-el31);
akufel(2,1)=c1×el15+c2×el33;
akufel(2,2)=-c1×el15+c4×el33;
akufel(2,3)=-c3×el15-c4×el33;
```

```
akufel(2,4)=c3×el15-c2×el33;
akufel(3,1)=0.25×(el15-el31);
akufel(3,2)=-0.25×(el15+el31);
akufel(3,3)=0.25×(-el15+el31);
akufel(3,4)=0.25×(el15+el31);
akufel(4,1)=-c1×el15+c4×el33;
akufel(4,2)=c1×el15+c2×el33;
akufel(4,3)=c3×el15-c2×el33;
akufel(4,4)=-c3×el15-c4×el33;
akufel(5,1)=-0.25×(el15+el31);
akufel(5,2)=0.25×(el15-el31);
akufel(5,3)=0.25×(el15+el31);
akufel(5,4)=0.25×(-el15+el31);
akufel(6,1)=-c3×el15-c4×el33;
akufel(6,2)=c3×el15-c2×el33;
akufel(6,3)=c1×el15+c2×el33;
akufel(6,4)=-c1×el15+c4×el33;
akufel(7,1)=0.25×(-el15+el31);
akufel(7,2)=0.25×(el15+el31);
akufel(7,3)=0.25×(el15-el31);
akufel(7,4)=-0.25×(el15+el31);
akufel(8,1)=c3×el15-c2×el33;
akufel(8,2)=-c3×el15-c4×el33;
akufel(8,3)=-c1×el15+c4×el33;
akufel(8,4)=c1×el15+c2×el33;
i=1;
while i ≤ 8
j=1;
while j ≤ 4
akufel(i,j)=t×akufel(i,j);
j=j+1;
end
i=i+1;
end
con=c1×eps11+c2×eps33;
i=1;
while i ≤ 4
akffel(i,i)=con;
i=i+1;
end
akffel(1,2)=-c1×eps11+c4×eps33;
```

```
akffel(1,3)=-c3×eps11-c4×eps33;
akffel(1,4)=c3×eps11-c2×eps33;
akffel(2,3)=c3×eps11-c2×eps33;
akffel(2,4)=-c3×eps11-c4×eps33;
akffel(3,4)=-c1×eps11+c4×eps33;
akffel(2,1)=akffel(1,2);
akffel(3,1)=akffel(1,3);
akffel(4,1)=akffel(1,4);
akffel(3,2)=akffel(2,3);
akffel(4,2)=akffel(2,4);
akffel(4,3)=akffel(3,4);
i=1;
while i ≤ 4
j=1;
while j ≤ 4
akffel(i,j)=t×akffel(i,j);
j=j+1;
end
i=i+1;
end
end
```

Function Elemt for Case 1

```
function [akutel,actuel,akttel]=elemt(a,b,t,al1,al3,al13,theta0,ak11,ak33)
c1=b/a/3;
c2=a/b/3;
c3=c1/2;
c4=c2/2;
con=c1× ak11+c2× ak33;
i=1;
while i ¡= 4
akttel(i,i)=con;
i=i+1;
end
akttel(1,2)=-c1× ak11+c4× ak33;
akttel(1,3)=-c3× ak11-c4× ak33;
akttel(1,4)=c3× ak11-c2× ak33;
akttel(2,3)=c3× ak11-c2× ak33;
akttel(2,4)=-c3× ak11-c4× ak33;
akttel(3,4)=-c1× ak11+c4× ak33;
akttel(2,1)=akttel(1,2);
akttel(3,1)=akttel(1,3);
akttel(4,1)=akttel(1,4);
akttel(3,2)=akttel(2,3);
akttel(4,2)=akttel(2,4);
akttel(4,3)=akttel(3,4);
akttel=t× akttel;
akutel(1,1)=-2× al1× b-2× al13× a;
akutel(2,1)=-2× al13× b-2× al3× a;
akutel(3,1)=2× al1× b-al13× a;
akutel(4,1)=2× al13× b-al3× a;
akutel(5,1)=al1× b+al13× a;
akutel(6,1)=al13× b+al3× a;
akutel(7,1)=-al1× b+2× al13× a;
akutel(8,1)=-al13× b+2× al3× a;
akutel(1,2)=-2× al1× b-al13× a;
akutel(2,2)=-2× al13× b-al3× a;
akutel(3,2)=2× al1× b-2× al13× a;
akutel(4,2)=2× al13× b-2× al3× a;
akutel(5,2)=al1× b+2× al13× a;
akutel(6,2)=al13× b+2× al3× a;
akutel(7,2)=-al1× b+al13× a;
akutel(8,2)=-al13× b+al3× a;
```

```
akutel(1,3)=-all× b-all3× a;
akutel(2,3)=-all3× b-al3× a;
akutel(3,3)=all× b-2× all3× a;
akutel(4,3)=all3× b-2× al3× a;
akutel(5,3)=2× all× b+2× all3× a;
akutel(6,3)=2× all3× b+2× al3× a;
akutel(7,3)=-2× all× b+all3× a;
akutel(8,3)=-2× all3× b+al3× a;
akutel(1,4)=-all× b-2× all3× a;
akutel(2,4)=-all3× b-2× al3× a;
akutel(3,4)=all× b-all3× a;
akutel(4,4)=all3× b-al3× a;
akutel(5,4)=2× all× b+all3× a;
akutel(6,4)=2× all3× b+al3× a;
akutel(7,4)=-2× all× b+2× all3× a;
akutel(8,4)=-2× all3× b+2× al3× a;
cut=t/6;
akutel=cut× akutel;
actuel=theta0× akutel';
end
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Bibliography

[1] Curie, P. and Curie, J. *Comptes Rendus*, volume 91. No. 294, 1880.

[2] Tzou, H.S. and Zhong, J.P. and Natori, M. Sensor mechanics of distributed shell convolving sensors applied to flexible rings. *Journal of Vibration and Acoustics*, 115(1):40–46, January 1969.

[3] Anderson, E.H. and Hagood, N.W. Simulataneous piezoelectric sensing/actuation: Analysis and application to controlled structures. *Journal of Sound and Vibration*, 174(5):617–639, 1996.

[4] Tzou, H.S. and Tseng, C.I. Advanced dynamic measurements for distributed mechanical systems. part 2: Finite element analysis. *Proceedings of the 1988 Symposium on Advanced Manufacturing, Lexington, KY*, pages 95–98, 1988.

[5] Uchino, K. Recent development of piezoelectric actuators for adaptive structures. *SPIE Conf. on Adaptive Structures, SPIE*, 2040:245–257, 1993.

[6] Rao, S.S. and Sunar, M. Piezoelectricity and its use in disturbance sensing and control of flexible structures. *Applied Mechanics Reviews*, 47:113–123, 1994.

[7] Yu, Y.Y. Some recent advances in linear and nonlinear dynamic modeling of elastic and piezoelectric plates. *Journal of Intelligent Material Systems and Structures*, 6(2):237–254, March 1995.

[8] G. Taguchi. *Off-line & On-line Quality Control System.* International Conference on Quality Control, Tokyo, Japan, 1978.

[9] G. Taguchi. *On-line Quality Control During Production.* Japanese Standards Association, Dearborn, MI, 1981.

[10] G. Taguchi and M.S. Phadke. Quality Engineering through Design Optimization. *Conference Record, GLOBECOM 84 Meeting, IEEE Communications Society*, pages 1106–1113, November 1984.

[11] G. Taguchi and S. Konishi. *Orthogonal Arrays and Linear Graphs.* American Supplier Institute, Dearborn, MI, 1987.

[12] Crawley, E.F. and DeLuis, J. Use of piezoelectric actuators as elements of intelligent structures. *AIAA Journal*, 10:1373–1385, 1987.

[13] Kawai, H. The piezoelectricity of poly(vinylidene fluoride). *Japanese Journal of Applied Physics*, 8(7):975–976, 1969.

[14] Cheng, S.J and Chim, D.C. and Lu, Y. and Shaikh, N. and Ianno, N.J. Smart structural composites with monofilament, piezoelectric sensors. *Journal of Intelligent Material Systems and Structures*, 6(3):436–442, 1995.

[15] Song, O. and Librescu, L. and Rogers, C.A. Application of adaptive technology to static aeroelastic control of wing structures. *AIAA Journal*, 30(12):2882–2889, December 1992.

[16] Zhou, Y.S. and Tiersten, H.F. Elastic analysis of laminated plates in cylindrical bending due to piezoelectric actuators. *Smart Materials and Structures*, 3(3):255–265, September 1994.

[17] Denoyer, K.K. and Kwak, M.K. Dynamic modeling and vibration suppression of a slowing structure utilizing piezoelectric sensors and actuators. *Journal of Sound and Vibration*, 189(1):13–31, 1996.

[18] Tzou, H.S. Active vibration control of distributed parameter systems by finite element method. *Computers in Engineering*, 3:599–604, 1988.

[19] Tzou, H.S. and Tseng, C.I. Distributed piezoelectric sensor/actuator design for dynamic measurement/control of distributed parameter systems: A piezoelectric finite element approach. *Sound and Vibration*, 137:91–94, 1990.

[20] Ha, S.K. and Chang, F.K. Finite element modeling of the response of laminated composites with distributed piezoelectric actuators. *AIAA/ASME/ASCE/AHS/ASC 31st Structures, Structural Dynamics and Materials Conference*, pages 2323–2330, 1990.

[21] Kagawa, Y. and Tsuchiya, T. and Kataoka, T and Yamabuchi, T. and Furakawa, T. Finite element simulation of dynamic responses of piezoelectric actuators. *Journal of Sound and Vibration*, pages 519–538, 1996.

[22] Sunar, M. and Rao, S.S. Distributed Modeling and Actuator Location for Piezoelectric Control Systems. *AIAA Journal*, 34:2209–2211, 1996.

[23] Mindlin, R.D. Equations of High Frequency Vibrations of Thermopiezoelectric Crystal Plates. *International Journal of Solids and Structures*, 10(6):625–637, 1996.

[24] Rao, S.S. and Sunar, M. Analysis of distributed thermopiezoelectric sensors and actuators in advanced intelligent structures. *AIAA Journal*, 31:1280–1286, 1993.

[25] Tzou, H.S. and Ye, R. Piezothermoelasticity and Precision Control of Piezoelectric Systems: Theory and Finite Element Analysis. *Journal of Vibration and Acoustics*, 116(4):489–495, 1994.

[26] Shang, J.S. Robust Design and Optimization of Material Handling in an FMS. *International Journal of Production Research*, 33:2437–2454, 1995.

[27] Rowlands, H. and Pham, D.T. Application of Taguchi Method to the Design of a Robot Sensor. *Robotica*, 6:607–617, 1995.

[28] Mertol, A. Application of the Taguchi Method on the Robust Design of Molded 225 Plastic Ball Greid Array Packages. *IEEE Transactions on Components, Packaging, and Manufacturing*, 18:734–743, November 1995.

[29] Phadke, M.S. *Quality Engineering Using Robust Design.* AT&T Bell Laboratories. Prentice Hall, Englewood Cliffs, New Jersey, ist edition, 1989.

[30] Park, G.J. and Hwang, W.J. and Lee, W. Structural Optimization Post-Process using Taguchi Method. *JSME International Journal, Series A: Mechanics and Material*, 37:166–172, Apr 1994.

[31] Guyan, R.J. Reduction of Stiffness and Mass Matrices. *AIAA Journal*, 3:380–386, May 1965.

[32] Nowacki, W. Some General Theorems of Thermopiezoelectricity. *Journal of Thermal Stresses*, 1(2):171–182, 1978.

[33] Tiersten, H.F. *Linear Piezoelectric Plate Vibrations.* Plenum Press, New York, 1969.

[34] Condra, Lloyd.W. *Reliability Improvement with Design of Experiments.* AT&T Bell Laboratories. Marcel Dekker, Inc. New Jersey, ist edition, 1993.

[35] Kwakernaak, H. *Linear Optimal Control Systems.* John Wiley and Sons, New York, ist edition, 1972.

[36] Rao, S.S. Combined Structural and Control Optimization of Flexible Structures. *Engineering Optimization*, 13:1–16, 1988.

[37] *Matlab, Version 4.2c, The Mathwork Inc.* South Natick, MA, USA.

[38] Soderkvist, J. Dynamic Behavior of a Piezoelectric Beam. *The Journal of the Acoustical Society of America*, 90:686–692, August 1991.

[39] M.S. Phadke and G. Taguchi. Quality Engineering through Design Optimization. *Conference Record, GLOBECOM 87 Meeting, IEEE Communications Society*, pages 1002–1007, November 1987.

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